Question Examples from Advance Information for Paper 2

Proof by counter example

Give a counter-example to prove that each of the following statements is false.

a If $a^2 - b^2 > 0$, where a and b are real, then $a - b > 0$.

b There are no prime numbers divisible by 7.

c If x and y are irrational and $x \neq y$, then xy is irrational.

d For all real values of x, $\cos(90 - |x|)° = \sin x°$.

b $(3^n + 2)$ is prime for all positive integer values of n.

c $\sqrt{n}$ is irrational for all positive integers n.

For each statement, find a counter-example to prove that it is false.

If a and b are positive integers and $a \neq b$, then $\log_a b$ is irrational.

For all real values of x and y, $x^2 - 2y(x - y) \geq 0$.

Prove, by counter-example, that each of the following statements is false.

a For all positive real values of x, $\sqrt[3]{x} \leq x$. **(2)**

b For all positive integer values of n, $(n^3 - n + 7)$ is prime. **(2)**

c Prove, by counter-example, that the statement

"if a is rational and b is irrational then $\log_a b$ is irrational"

is false. **(2)**

For each statement, either prove that it is true or find a counter-example to prove that it is false.

a If a and b are irrational and $a \neq b$, then $(a + b)$ is irrational. **(2)**

Proof by exhaustion (you need to take every case that might happen and show that the statement is true every time, for example all odd numbers and all even numbers etc.)

Prove that $n^7 - n$ is divisible by 7 if n is a positive integer.

Show that $m^2 - 1$ is multiple of 3 if m is not multiple of 3.

Show that the cube numbers of 3 to 7 are multiples of 9 or 1 more or 1 less than a multiple of 9.

Transformations of graphs, sketch curves defined by simple examples (this also came up in paper 1 so these examples are the same)

a Express $x^2 - 8x + 18$ in the form $(x + a)^2 + b$.

b Find the distance of the vertex of the curve $y = x^2 - 8x + 18$ from the origin, giving your answer in the form $k\sqrt{5}$.

c Describe two transformations that would map the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 18$.

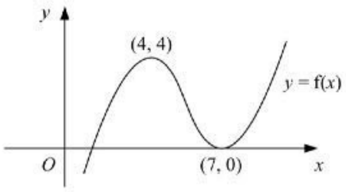
$$f(x) \equiv |2x + 5|, \ x \in \mathbb{R}.$$

a Sketch the graph $y = f(x)$, showing the coordinates of any points where the graph meets the coordinate axes.

b Evaluate $ff(-4)$.

$$g(x) \equiv f(x + k), \ x \in \mathbb{R}.$$

c State the value of the constant k for which $g(x)$ is symmetrical about the y-axis.



The diagram shows the curve with equation $y = f(x)$ which has two stationary points with coordinates (4, 4) and (7, 0).

Showing the coordinates of any stationary points, sketch on separate diagrams the curves

a $y = 1 + 2f(x)$,

b $y = f(-3x)$.

The function f is defined by

$$f : x \to x^{\frac{1}{2}} - 2, \ x \in \mathbb{R}, \ x \geq 0.$$

Showing the coordinates of any points where each graph meets the coordinate axes, sketch on separate diagrams the graphs of

a $y = f(x)$,

b $y = 2 + |f(x)|$,

c $y = 3f(x + 1)$.

$$f(x) \equiv \frac{3x - 5}{x - 2}, \ x \in \mathbb{R}, \ x \neq 2.$$

a Find $f^{-1}(x)$ and state its domain.

b Hence, or otherwise, solve the equation $f(x) = 4$.

c Find the values of a and b such that

$$f(x) = a + \frac{b}{x - 2}.$$

d Hence, describe two transformations that map the graph of $y = \frac{1}{x}$ onto the graph of $y = f(x)$.

a Express $2x^2 - 4x + 7$ in the form $a(x + b)^2 + c$.

b Hence, describe in order a sequence of transformations that would map the graph of $y = 2x^2 - 4x + 7$ onto the graph of $y = x^2$.

a Describe clearly, in order, the sequence of transformations that would map the graph of $y = \sqrt{x}$ onto the graph of $y = 2 - 3\sqrt{x}$.

b Sketch the graph of $y = 2 - 3\sqrt{x}$ showing the coordinates of any points where the graph meets the coordinate axes.