

Question Examples from Advance Information for Paper 2 (OCR)

Proof - as for paper 1

Prove, by counter-example, that each of the following statements is false.

- a** For all positive real values of x , $\sqrt[3]{x} \leq x$. (2)
b For all positive integer values of n , $(n^3 - n + 7)$ is prime. (2)

- a** Given that $n = 2m + 1$, find and simplify an expression in terms of m for $n^2 + 2n$. (1)
b Hence, use proof by contradiction to prove that if $(n^2 + 2n)$ is even, where n is an integer, then n is even. (5)

Use proof by contradiction to prove that there are no positive integers, x and y , such that

$$x^2 - y^2 = 1. \quad (6)$$

For each statement, either prove that it is true or find a counter-example to prove that it is false.

- a** If a and b are irrational and $a \neq b$, then $(a + b)$ is irrational. (2)
b If m and n are consecutive odd integers, then $(m + n)$ is divisible by 4. (3)
c For all real values of x , $\cos x \leq 1 + \sin x$. (2)

- a** Show that if $\log_2 3 = \frac{p}{q}$, then
$$2^p = 3^q. \quad (2)$$

b Use proof by contradiction to prove that $\log_2 3$ is irrational. (4)
c Prove, by counter-example, that the statement
“if a is rational and b is irrational then $\log_a b$ is irrational”
is false. (2)

Use proof by contradiction to prove each of the following statements.

- a** If n^3 is odd, where n is a positive integer, then n is odd.
b If x is irrational, then \sqrt{x} is irrational.
c If a , b and c are integers and bc is not divisible by a , then b is not divisible by a .
d If $(n^2 - 4n)$ is odd, where n is a positive integer, then n is odd.
e There are no positive integers, m and n , such that $m^2 - n^2 = 6$.

Given that x and y are integers and that $(x^2 + y^2)$ is divisible by 4, use proof by contradiction to prove that

- a** x and y are not both odd,
b x and y are both even.

a Prove that if

$$\sqrt{2} = \frac{p}{q},$$

where p and q are integers, then p must be even.

b Use proof by contradiction to prove that $\sqrt{2}$ is irrational.

Prove by contradiction that there is no greatest even positive integer. [3]

Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n . [4]

Prove that the sum of the squares of any two consecutive integers is of the form $4k + 1$, where k is an integer. [4]

- a Prove that the difference of the squares of two consecutive even numbers is always divisible by 4.
b Is this statement true for odd numbers? Give a reason for your answer.

Prove by contradiction that $\sqrt{2}$ is an irrational number.

Prove by contradiction that there are infinitely many prime numbers.

Solving equations; algebra techniques

Solve the equation $x^{\frac{2}{3}} - x^{\frac{1}{3}} - 6 = 0$. [5]

Find the set of values of k for which the equation $x^2 + 2x + 11 = k(2x - 1)$ has two distinct real roots. [7]

- i. Express $4 + 12x - 2x^2$ in the form $a(x + b)^2 + c$. [4]
- ii. State the coordinates of the maximum point of the curve $y = 4 + 12x - 2x^2$. [2]

