

Question Examples from Advance Information for Paper 1

Factor theorem, cubic graphs (also sketch all the cubic graphs in the questions below!)

The curve C has equation

$$y = (x + 3)(x - 1)^2.$$

(a) Sketch C showing clearly the coordinates of the points where the curve meets the coordinate axes. (4)

(b) Show that the equation of C can be written in the form

$$y = x^3 + x^2 - 5x + k,$$

where k is a positive integer, and state the value of k . (2)

There are two points on C where the gradient of the tangent to C is equal to 3.

(c) Find the x -coordinates of these two points. (6)

(Total 12 marks)

$$f(x) = 2x^3 - 7x^2 - 5x + 4$$

(a) Find the remainder when $f(x)$ is divided by $(x - 1)$. (2)

(b) Use the factor theorem to show that $(x + 1)$ is a factor of $f(x)$. (2)

(c) Factorise $f(x)$ completely. (4)

(Total 8 marks)

(a) The polynomial $p(x)$ is defined by $p(x) = 6x^3 - 19x^2 + 9x + 10$.

(i) Find $p(2)$. (1 mark)

(ii) Use the Factor Theorem to show that $(2x + 1)$ is a factor of $p(x)$. (3 marks)

(iii) Write $p(x)$ as the product of three linear factors. (2 marks)

(b) Hence simplify $\frac{3x^2 - 6x}{6x^3 - 19x^2 + 9x + 10}$. (2 marks)

$$f(x) \equiv x^3 - 5x^2 + ax + b.$$

Given that $(x + 2)$ and $(x - 3)$ are factors of $f(x)$,

a show that $a = -2$ and find the value of b .

b Hence, express $f(x)$ as the product of three linear factors.

$$f(x) \equiv 3x^3 - x^2 - 12x + 4.$$

a Show that $(x - 2)$ is a factor of $f(x)$.

b Solve the equation $f(x) = 0$.

$$f(x) \equiv x^3 - 2x^2 - 11x + 12.$$

- a** Show that $(x - 1)$ is a factor of $f(x)$.
- b** Hence, express $f(x)$ as the product of three linear factors.

$$f(x) \equiv 6x^3 - 7x^2 - 71x + 12.$$

Given that $f(4) = 0$, find all solutions to the equation $f(x) = 0$.

$$g(x) \equiv x^3 + 7x^2 + 7x - 6.$$

Given that $x = -2$ is a solution to the equation $g(x) = 0$,

- a** express $g(x)$ as the product of a linear factor and a quadratic factor,
- b** find, to 2 decimal places, the other two solutions to the equation $g(x) = 0$.

$$f(x) \equiv 2x^3 - x^2 - 15x + c.$$

Given that $(x - 2)$ is a factor of $f(x)$,

- a** find the value of the constant c ,
- b** fully factorise $f(x)$.

$$g(x) \equiv x^3 + px^2 - 13x + q.$$

Given that $(x + 1)$ and $(x - 3)$ are factors of $g(x)$,

- a** show that $p = 3$ and find the value of q ,
- b** solve the equation $g(x) = 0$.

Transformations of Graphs (be careful of the order of successive transformations if they are in the same direction!)