Question Examples from Advance Information for Paper 3

Proof by contraction

a Given that $n = 2m + 1$, find and simplify an expression in terms of m for $n^2 + 2n$. **(1)**

b Hence, use proof by contradiction to prove that if $(n^2 + 2n)$ is even, where n is an integer, then n is even. **(5)**

Use proof by contradiction to prove that there are no positive integers, x and y, such that

$$x^2 - y^2 = 1.$$ **(6)**

a Show that if $\log_2 3 = \frac{p}{q}$, then

$$2^p = 3^q.$$ **(2)**

b Use proof by contradiction to prove that $\log_2 3$ is irrational. **(4)**

c Prove, by counter-example, that the statement

"if a is rational and b is irrational then $\log_a b$ is irrational"

is false. **(2)**

Use proof by contradiction to prove each of the following statements.

a If n^3 is odd, where n is a positive integer, then n is odd.

b If x is irrational, then $\sqrt{x}$ is irrational.

c If a, b and c are integers and bc is not divisible by a, then b is not divisible by a.

d If $(n^2 - 4n)$ is odd, where n is a positive integer, then n is odd.

e There are no positive integers, m and n, such that $m^2 - n^2 = 6$.

Given that x and y are integers and that $(x^2 + y^2)$ is divisible by 4, use proof by contradiction to prove that

a x and y are not both odd,

b x and y are both even.

a Prove that if

$$\sqrt{2} = \frac{p}{q},$$

where p and q are integers, then p must be even.

b Use proof by contradiction to prove that $\sqrt{2}$ is irrational.

Prove by contradiction that there is no greatest even positive integer. [3]

Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers n. [4]

Prove that the sum of the squares of any two consecutive integers is of the form $4k + 1$, where k is an integer. [4]

Prove by contradiction that $\sqrt{2}$ is an irrational number.

a Prove that the difference of the squares of two consecutive even numbers is always divisible by 4.

b Is this statement true for odd numbers? Give a reason for your answer.

Prove by contradiction that there are infinitely many prime numbers.

Proof by contradiction