Question Examples from Advance Information for Paper 3 (OCR)

Modulus function

Solve $|5x-2| = -\frac{1}{4}x + 8$. **(4 marks)**

a On the same set of axes, sketch $y = |12 - 5x|$ and $y = -2x + 3$. **(3 marks)**

b State with a reason whether there are any solutions to the equation
$|12 - 5x| = -2x + 3$ **(2 marks)**

The function f is defined by

$$f : x \to |3x - a|, \ x \in \mathbb{R}.$$

where a is a positive constant.

a Find $ff(-2a)$. **(2)**

b Sketch the graph $y = f(x)$, showing the coordinates of any points where the graph meets the coordinate axes. **(3)**

c Solve the equation $f(x) = x$, giving your answers in terms of a. **(3)**

a Sketch on the same set of axes the graphs of $y = |x|$ and $y = |2x-3|$. **(3)**

b Hence, or otherwise, solve the equation

$$|x| = |2x-3|.$$ **(4)**

(a) Sketch and label on the same set of axes the graphs of:

(i) $y = |x|$; *(1 mark)*

(ii) $y = |2x-4|$. *(2 marks)*

(b) (i) Solve the equation $|x| = |2x-4|$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|x| > |2x-4|$. *(2 marks)*

(a) Sketch the graph of $y = |2x|$. *(1 mark)*

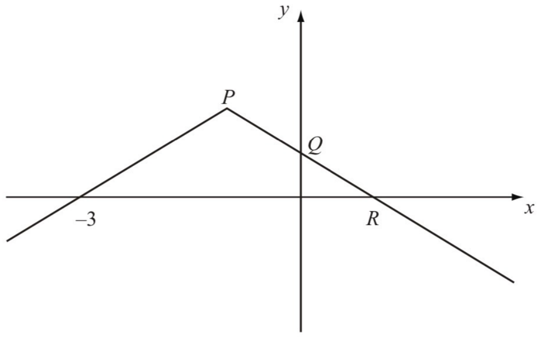
(b) On a separate diagram, sketch the graph of $y = 4 - |2x|$, indicating the coordinates of the points where the graph crosses the coordinate axes. *(3 marks)*

(c) Solve $4 - |2x| = x$. *(3 marks)*

(d) Hence, or otherwise, solve the inequality $4 - |2x| > x$. *(2 marks)*

The equation $|2x - 11| = \frac{1}{2}x + k$ has exactly two distinct solutions.

Find the range of possible values of k. **(4 marks)**



The diagram above shows the graph of $y = \mathrm{f}(x),\ x \in \Re$.
The graph consists of two line segments that meet at the point P.
The graph cuts the y-axis at the point Q and the x-axis at the points (–3, 0) and R.

Sketch, on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(2)

(b) $y = \mathrm{f}(-x)$.

(2)

Given that $\mathrm{f}(x) = 2 - |x + 1|$,

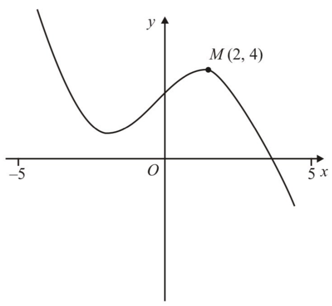
(c) find the coordinates of the points P, Q and R,

(3)

(d) solve $\mathrm{f}(x) = \frac{1}{2}x$.

(5)
(Total 12 marks)

Functions: transformations and inverses



The figure above shows the graph of $y = f(x)$, $-5 \le x \le 5$.
The point M (2, 4) is the maximum turning point of the graph.

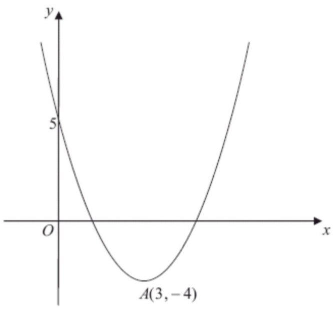
Sketch, on separate diagrams, the graphs of

(a) $y = \mathrm{f}(x) + 3$,

(2)

(b) $y = |\mathrm{f}(x)|$,

(2)



The diagram above shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f\left(\frac{1}{2}x\right)$.

(4)

(b) Sketch the curve with equation

$$y = f(|x|)$$

(3)

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

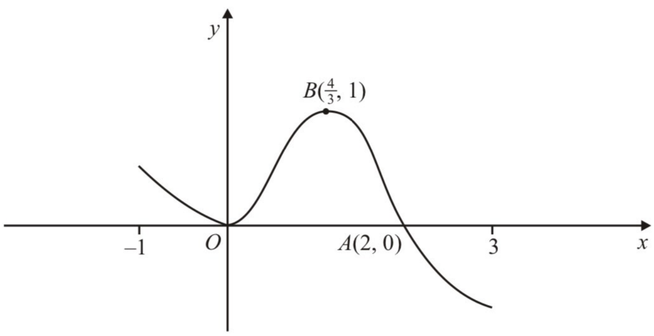
The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)
(Total 10 marks)



The diagram above shows a sketch of the curve with equation $y = f(x)$, $-1 \le x \le 3$. The curve touches the x-axis at the origin O, crosses the x-axis at the point $A(2, 0)$ and has a maximum at the point $B(\frac{4}{3}, 1)$.

In separate diagrams, show a sketch of the curve with equation

(a) $y = f(x + 1)$,

(3)

(b) $y = |f(x)|$,

(3)

(c) $y = f(|x|)$,

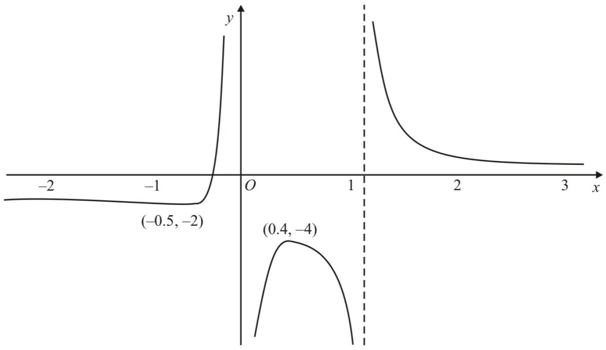
(4)

marking on each sketch the coordinates of points at which the curve

(i) has a turning point,

(ii) meets the x-axis.

(Total 10 marks)



The diagram above shows a sketch of part of the curve with equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a minimum point at (–0.5, –2) and a maximum point at (0.4, –4). The lines $x = 1$, the x-axis and the y-axis are asymptotes of the curve, as shown in the diagram above.

On a separate diagram sketch the graphs of

(a) $y = |f(x)|$,

(4)

(b) $y = f(x - 3)$,

(4)

(c) $y = f(|x|)$.

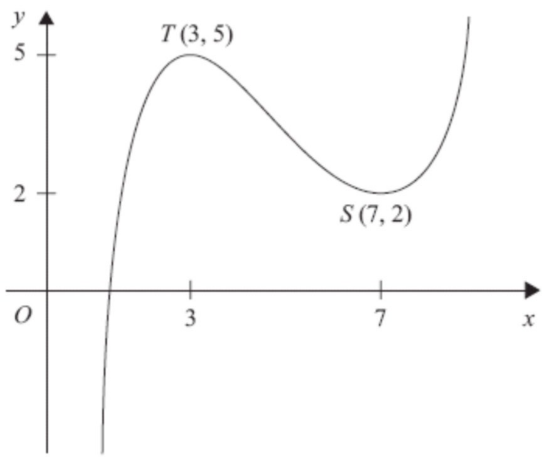
(4)

In each case show clearly

(i) the coordinates of any points at which the curve has a maximum or minimum point,

(ii) how the curve approaches the asymptotes of the curve.

(Total 12 marks)



The figure above shows the graph of $y = f(x)$, $1 < x < 9$.
The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$,

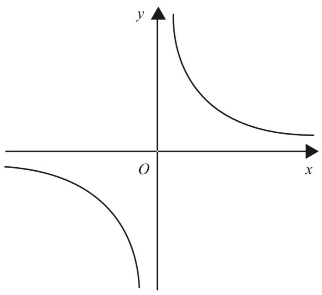
(3)

(b) $y = |f(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.

(Total 6 marks)



The diagram above shows a sketch of the curve with equation $y = \frac{3}{x}$, $x \neq 0$.

(a) On a separate diagram, sketch the curve with equation $y = \frac{3}{x+2}$, $x \neq -2$, showing the coordinates of any point at which the curve crosses a coordinate axis.

(3)

(b) Write down the equations of the asymptotes of the curve in part (a).

(2)
(Total 5 marks)

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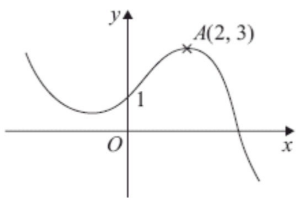


The diagram above shows a sketch of the graph of $y = f(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = f(-x) + 1$,

(ii) $y = f(x + 2) + 3$,

(iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

(Total 9 marks)

$$f(x) = \frac{2x+5}{x+3} - \frac{1}{(x+3)(x+2)}, \quad x > -2.$$

(a) Express f(x) as a single fraction in its simplest form.

(5)

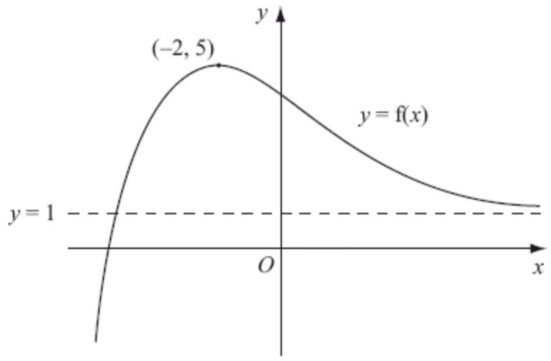
(b) Hence show that $f(x) = 2 - \frac{1}{x+2}$, $x > -2$.

(2)

The curve $y = \frac{1}{x}$, $x > 0$, is mapped onto the curve $y = f(x)$, using three successive transformations T_1, T_2 and T_3, where T_1 and T_3 are translations.

(c) Describe fully T_1, T_2 and T_3.

(4)
(Total 11 marks)



The diagram above shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$.

The curve has a maximum point $(-2, 5)$ and an asymptote $y = 1$, as shown in the diagram.

On separate diagrams, sketch the curve with equation

(a) $y = \mathrm{f}(x) + 2$ **(2)**

(b) $y = 4\mathrm{f}(x)$ **(2)**

(c) $y = \mathrm{f}(x + 1)$ **(3)**

On each diagram, show clearly the coordinates of the maximum point and the equation of the asymptote.

$$\mathrm{f} : x \rightarrow 2 + \log_4 x,\ x \in \mathbb{R},\ x > 0.$$

a Evaluate ff(1). **(3)**

b Solve the equation $\mathrm{f}(x) = 0$. **(2)**

c Find the inverse function $\mathrm{f}^{-1}(x)$. **(3)**

The function f is given by

$$\mathrm{f} : x \rightarrow \mathrm{e}^{\frac{1}{2}x} - 3,\ x \in \mathbb{R}.$$

a Find $\mathrm{f}^{-1}(x)$ and state its domain. **(4)**

b Sketch the curve $y = \mathrm{f}^{-1}(x)$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

The function g is given by

$$\mathrm{g} : x \rightarrow \ln (x + 5),\ x \in \mathbb{R},\ x > -5.$$

c Evaluate fg(4). **(2)**

d Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{g}(x)$. **(4)**

The functions f and g are defined by

$$\mathrm{f} : x \rightarrow x^2 + 4,\ x \in \mathbb{R},$$

$$\mathrm{g} : x \rightarrow 2x - \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0.$$

a Evaluate gf(−2). **(2)**

b Find and simplify an expression for fg(x). **(3)**

c Find the values of x for which $\mathrm{fg}(x) = 5$. **(4)**

The functions f and g are defined by

$$f : x \rightarrow kx + 2, \; x \in \mathbb{R},$$

$$g : x \rightarrow x - 3k, \; x \in \mathbb{R},$$

where k is a constant.

a Find expressions in terms of k for

i $f^{-1}(x)$,

ii $fg(x)$. **(4)**

Given that $fg(7) = 4$,

b find the two possible values of k. **(3)**

The function f is defined by

$$f : x \rightarrow \frac{x+b}{x+a}, \; x \in \mathbb{R}, \; x \neq 2.$$

a State the value of the constant a.

Given that $f(6) = 4$,

b find the value of the constant b,

c find $f^{-1}(x)$ and state its domain.

The functions f and g are defined by

$$f : x \rightarrow x^2 - 3x, \; x \in \mathbb{R}, \; x \geq 1\tfrac{1}{2},$$

$$g : x \rightarrow 2x + 3, \; x \in \mathbb{R}.$$

a Find, in the form $f^{-1} : x \rightarrow \ldots$, the inverse function of f and state its domain.

b On the same set of axes, sketch $y = f(x)$ and $y = f^{-1}(x)$.

Given that $f^{-1}g^{-1}(12) = a(1 + \sqrt{3})$,

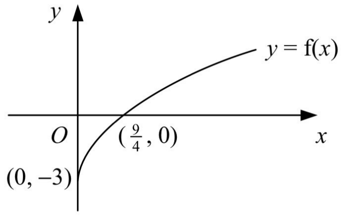
c show that $a = 1\tfrac{1}{2}$.

$$f : x \rightarrow 2 + \log_4 x, \; x \in \mathbb{R}, \; x > 0.$$

a Evaluate $ff(1)$. **(3)**

b Solve the equation $f(x) = 0$. **(2)**

c Find the inverse function $f^{-1}(x)$. **(3)**



The diagram shows the graph of $y = f(x)$ which meets the x-axis at the point $(\frac{9}{4}, 0)$ and the y-axis at the point $(0, -3)$.

a Sketch on separate diagrams the graphs of

i $y = |f(x)|$,

ii $y = f^{-1}(x)$. **(4)**

Given that $f(x)$ is of the form $f(x) \equiv ax^{\frac{1}{2}} + b, \; x \in \mathbb{R}, \; x \geq 0$,

b find the values of the constants a and b, **(3)**

c find an expression for $f^{-1}(x)$. **(3)**

The functions f and g are defined by

$$f : x \rightarrow kx + 2, \ x \in \mathbb{R},$$

$$g : x \rightarrow x - 3k, \ x \in \mathbb{R},$$

where k is a constant.

a Find expressions in terms of k for

i $f^{-1}(x)$,

ii $fg(x)$. **(4)**

Given that $fg(7) = 4$,

b find the two possible values of k. **(3)**

The function f is given by

$$f : x \rightarrow e^{\frac{1}{2}x} - 3, \ x \in \mathbb{R}.$$

a Find $f^{-1}(x)$ and state its domain. **(4)**

b Sketch the curve $y = f^{-1}(x)$, showing the coordinates of any points of intersection with the coordinate axes. **(3)**

The function g is given by

$$g : x \rightarrow \ln(x + 5), \ x \in \mathbb{R}, \ x > -5.$$

c Evaluate $fg(4)$. **(2)**

d Solve the equation $f^{-1}(x) = g(x)$. **(4)**

The function f is defined by

$$f : x \rightarrow 3 + \ln(x + 2), \ x \in \mathbb{R}, \ x \geq k,$$

where k is a constant.

Given that the range of f is $f(x) \geq 3$,

a find the value of k, **(3)**

b find $f^{-1}(x)$, stating its domain clearly. **(4)**

The function g is defined by

$$g : x \rightarrow 4 + \ln(x - 1), \ x \in \mathbb{R}, \ x > 1.$$

c Find, in terms of e, the value of x such that $f(x) = g(x)$. **(4)**

The functions f and g are given by

$$f(x) \equiv \frac{x}{x+2}, \ x \in \mathbb{R}, \ x \neq -2,$$

$$g(x) \equiv \frac{3}{x}, \ x \in \mathbb{R}, \ x \neq 0$$

a Solve the equation $fg(x) = 4$. **(4)**

b Find $f^{-1}(x)$, stating its domain clearly. **(4)**

c Solve the equation $f(x) = f^{-1}(x)$. **(3)**

The function f is defined by

$$f(x) \equiv x^2 - 2x - 9,\ x \in \mathbb{R},\ x \geq k.$$

a Find the minimum value of the constant k for which $f^{-1}(x)$ exists. **(3)**

Given that k takes the value found in part **a**,

b solve the equation $f^{-1}(x) = 4$, **(2)**

c sketch the curve $y = |f(x)|$, **(3)**

d find the values of x for which $|f(x)| = 6$. **(5)**

The function f is defined by

$$f : x \to 2 - \frac{3}{x},\ x \in \mathbb{R},\ x \neq 0.$$

a Find the value of ff(1). **(2)**

b Find $f^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$g : x \to x^2,\ x \in \mathbb{R}.$$

c Solve the equation $gf(x) = 1$. **(4)**

The function f is defined by

$$f : x \to e^{\frac{1}{2}x} - 2,\ x \in \mathbb{R}.$$

a Evaluate f(ln 9). **(2)**

b State the range of f. **(1)**

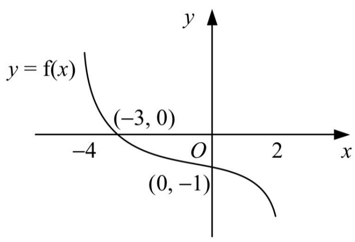
c Find $f^{-1}(x)$ and state its domain. **(4)**

The function g is defined by

$$g : x \to x^2 + 4x,\ x \in \mathbb{R}.$$

d Find and simplify an expression for $gf(x)$. **(3)**

e Solve the equation $gf(x) + 1 = 0$. **(2)**



The diagram shows the curve $y = f(x)$. The domain of f is $-4 \leq x \leq 2$ and the curve intersects the coordinate axes at the points (–3, 0) and (0, –1).

a Explain how the graph shows that f is one-one. **(1)**

b Showing the coordinates of any points of intersection with the axes, sketch on separate diagrams the graphs of

i $y = |f(x)|$,

ii $y = f^{-1}(x)$. **(5)**

$$f(x) \equiv \frac{5}{(x+1)(2x-3)} + \frac{1}{x+1},\ x \in \mathbb{R},\ x \geq 2.$$

a Show that $f(x) = \dfrac{2}{2x-3}$. **(4)**

b Find the range of f. **(2)**

c Find an expression for $f^{-1}(x)$. **(3)**

$$g(x) \equiv \frac{1}{x-2},\ x \in \mathbb{R},\ x \neq 2.$$

d Solve the equation $fg(x) = \frac{2}{3}$. **(4)**

Investigation of curves; differentiation; Newton-Raphson method

$$f(x) = 4\cos x + e^{-x}.$$

(a) Show that the equation $f(x) = 0$ has a root α between 1.6 and 1.7 (2)

(b) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to obtain a second approximation to α. Give your answer to 3 significant figures. (4)

(Total 6 marks)

$$f(x) = 3\sqrt{x} + \frac{18}{\sqrt{x}} - 20$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval [1.1, 1.2]. (2)

(b) Find $f'(x)$. (3)

(c) Using $x_0 = 1.1$ as a first approximation to α, apply the Newton-Raphson procedure once to $f(x)$ to find a second approximation to α, giving your answer to 3 significant figures. (4)

(Total 9 marks)

$$f(x) = x^2 + \frac{3}{4\sqrt{x}} - 3x - 7, \quad x > 0$$

A root α of the equation $f(x) = 0$ lies in the interval [3, 5].

Taking 4 as a first approximation to α, apply the Newton-Raphson process once to $f(x)$ to obtain a second approximation to α. Give your answer to 2 decimal places. (5)

(Total 5 marks)

The curve with equation $y = \frac{1}{2}x^2 - 3\ln x$, $x > 0$, has a stationary point at A.

a Find the exact x-coordinate of A. **(3)**

b Determine the nature of the stationary point. **(2)**

c Show that the y-coordinate of A is $\frac{3}{2}(1 - \ln 3)$. **(2)**

d Find an equation for the tangent to the curve at the point where $x = 1$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(3)**

A curve has the equation $y = e^x(x - 1)^2$.

a Find $\frac{dy}{dx}$. **(3)**

b Show that $\frac{d^2y}{dx^2} = e^x(x^2 + 2x - 1)$. **(2)**

c Find the exact coordinates of the turning points of the curve and determine their nature. **(4)**

d Show that the tangent to the curve at the point where $x = 2$ has the equation

$$y = e^2(3x - 5).$$ **(3)**

$$f(x) = \frac{6x}{(x-1)(x+2)} - \frac{2}{x-1}.$$

a Show that $f(x) = \frac{4}{x+2}$. **(5)**

b Find an equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate 2, giving your answer in the form $ax + by = c$, where a, b and c are integers. **(4)**

A curve has the equation

$$2\sin x - \tan 2y = 0.$$

a Show that $\frac{dy}{dx} = \cos x \cos^2 2y$. **(4)**

b Find an equation for the tangent to the curve at the point $(\frac{\pi}{3}, \frac{\pi}{6})$, giving your answer in the form $ax + by + c = 0$. **(3)**

Find an equation for the tangent to the curve with equation

$$y = (3 - x)^{\frac{3}{2}}$$

at the point on the curve with x-coordinate -1. **(4)**

a Sketch the curve with equation $y = 3 - \ln 2x$. **(2)**

b Find the exact coordinates of the point where the curve crosses the x-axis. **(2)**

c Find an equation for the tangent to the curve at the point on the curve where $x = 5$. **(4)**

This tangent cuts the x-axis at A and the y-axis at B.

d Show that the area of triangle OAB, where O is the origin, is approximately 7.20 **(3)**

Differentiate with respect to x

a $(3x - 1)^4$, **(2)**

b $\frac{x^2}{\sin 2x}$. **(3)**

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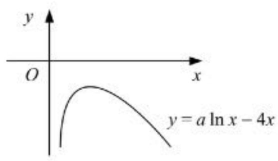


The diagram shows the curve with equation $y = a \ln x - 4x$, where a is a positive constant.

Find, in terms of a,

a the coordinates of the stationary point on the curve, **(4)**

b an equation for the tangent to the curve at the point where $x = 1$. **(3)**

Given that this tangent meets the x-axis at the point $(3, 0)$,

c show that $a = 6$. **(2)**

A curve has the equation $y = x^3 + 3x^2 - 16x + 2$.

a Find an equation of the tangent to the curve at the point $P\,(2, -10)$.

The tangent to the curve at the point Q is parallel to the tangent at the point P.

b Find the coordinates of the point Q.

A curve has the equation $y = 4^x$.

Show that the tangent to the curve at the point where $x = 1$ has the equation

$$y = 4 + 8(x - 1)\ln 2.$$ **(4)**

(b) Given that $y = \frac{3x+1}{2x+1}$, show that $\frac{dy}{dx} = \frac{1}{(2x+1)^2}$. *(3 marks)*

(a) Find $\frac{dy}{dx}$ when $y = (3x - 1)^{10}$. *(2 marks)*

(a) Find $\frac{dy}{dx}$ when:

(i) $y = (4x^2 + 3x + 2)^{10}$; *(2 marks)*

Coordinate geometry of circles

7 A circle has equation $x^2 + y^2 - 4x - 14 = 0$.

(a) Find:

(i) the coordinates of the centre of the circle; *(3 marks)*

(ii) the radius of the circle in the form $p\sqrt{2}$, where p is an integer. *(3 marks)*

(b) A chord of the circle has length 8. Find the perpendicular distance from the centre of the circle to this chord. *(3 marks)*

(c) A line has equation $y = 2k - x$, where k is a constant.

(i) Show that the x-coordinate of any point of intersection of the line and the circle satisfies the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$ *(3 marks)*

(ii) Find the values of k for which the equation

$$x^2 - 2(k+1)x + 2k^2 - 7 = 0$$

has equal roots. *(4 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (c)(ii). *(1 mark)*

A circle with centre C has equation $x^2 + y^2 + 2x - 12y + 12 = 0$.

(a) By completing the square, express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C; *(1 mark)*

(ii) the radius of the circle. *(1 mark)*

(c) Show that the circle does **not** intersect the x-axis. *(2 marks)*

(d) The line with equation $x + y = 4$ intersects the circle at the points P and Q.

(i) Show that the x-coordinates of P and Q satisfy the equation

$$x^2 + 3x - 10 = 0$$ *(3 marks)*

(ii) Given that P has coordinates $(2, 2)$, find the coordinates of Q. *(2 marks)*

(iii) Hence find the coordinates of the midpoint of PQ. *(2 marks)*

The circle C has equation $x^2 + y^2 - 8x - 16y + 72 = 0$.

a Find the coordinates of the centre and the radius of C. **(3)**

b Find the distance of the centre of C from the origin in the form $k\sqrt{5}$. **(2)**

The point A lies on C and the tangent to C at A passes through the origin O.

c Show that $OA = 6\sqrt{2}$. **(3)**

The circle C has equation $x^2 + y^2 - 4x - 6 = 0$ and the line l has equation $y = 3x - 6$.

a Show that l passes through the centre of C. **(3)**

b Find an equation for each tangent to C that is parallel to l. **(6)**

A circle with centre C has equation $(x+3)^2 + (y-2)^2 = 25$.

(a) Write down:

(i) the coordinates of C; *(2 marks)*

(ii) the radius of the circle. *(1 mark)*

(b) (i) Verify that the point $N(0, -2)$ lies on the circle. *(1 mark)*

(ii) Sketch the circle. *(2 marks)*

(iii) Find an equation of the normal to the circle at the point N. *(3 marks)*

(c) The point P has coordinates (2, 6).

(i) Find the distance PC, leaving your answer in surd form. *(2 marks)*

(ii) Find the length of a tangent drawn from P to the circle. *(3 marks)*

A circle has the equation $x^2 + y^2 - 6x + 2y - 15 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Find the radius of the circle. **(1)**

c Show that the tangent to the circle at the point (7, 2) has equation

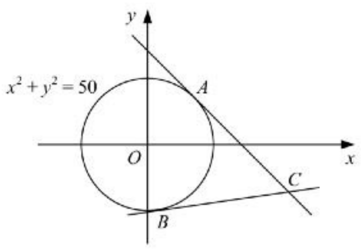
$$4x + 3y - 34 = 0.$$ **(4)**

A circle has the equation $x^2 + y^2 + 6x - 8y + 21 = 0$.

a Find the coordinates of the centre and the radius of the circle. **(3)**

The point P lies on the circle.

b Find the greatest distance of P from the origin. **(2)**



The diagram shows the circle with equation $x^2 + y^2 = 50$ and the tangents to the circle at the points A (5, 5) and B (1, −7).

a Find an equation of the tangent to the circle at A. **(3)**

b Show that the tangent to the circle at B has the equation

$$x - 7y - 50 = 0.$$ **(3)**

c Find the coordinates of the point C where the tangents to the circle at A and B intersect. **(2)**

The circle C has the equation $x^2 + y^2 + 2x - 14y + 30 = 0$.

a Find the coordinates of the centre of C. **(2)**

b Find the radius of C, giving your answer in the form $k\sqrt{5}$. **(2)**

c Show that the line $y = 2x - 1$ is a tangent to C and find the coordinates of the point of contact. **(4)**

The circle C has equation $x^2 + y^2 - 6x - 12y + 28 = 0$.

a Find the coordinates of the centre of C. **(2)**

The line $y = x - 2$ intersects C at the points A and B.

b Find the length AB in the form $k\sqrt{2}$. **(6)**

The circle C has centre $(8, -1)$ and passes through the point $(4, 1)$.

a Find an equation for C. **(3)**

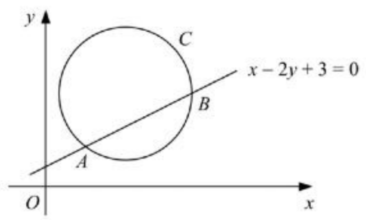
b Show that the line with equation $x + 2y + 4 = 0$ is a tangent to C. **(3)**

A circle has the equation $x^2 + y^2 - 2x - 7y - 16 = 0$.

a Find the coordinates of the centre of the circle. **(2)**

b Show that the radius of the circle is $k\sqrt{13}$, where k is an exact fraction to be found. **(2)**

c Find an equation of the tangent to the circle at the point $(4, 8)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. **(4)**



The line with equation $x - 2y + 3 = 0$ intersects the circle C at the points A and B as shown in the diagram above. Given that the centre of C has coordinates $(6, 7)$,

a find the coordinates of the mid-point of the chord AB. **(6)**

Given also that the x-coordinate of the point A is 3,

b find the coordinates of the point B, **(3)**

c find an equation for C. **(2)**

Arithmetic and geometric sequences

The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a for their first day, £($a + d$) for their second day, £($a + 2d$) for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d.

(2)

A picker who works for all 30 days will earn a total of £1005

(b) Show that $15(a + 40.75) = 1005$

(2)

(c) Hence find the value of a and the value of d.

(4)
(Total 8 marks)

Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and so on, so that the amounts of money she gave each year formed an arithmetic sequence.

(a) Find the amount of money she gave in Year 10.

(2)

(b) Calculate the total amount of money she gave over the 20-year period.

(3)

Kevin also gave money to the charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(c) Calculate the value of A.

(4)
(Total 9 marks)

A 40-year building programme for new houses began in Oldtown in the year 1951 (Year 1) and finished in 1990 (Year 40).

The numbers of houses built each year form an arithmetic sequence with first term a and common difference d.

Given that 2400 new houses were built in 1960 and 600 new houses were built in 1990, find

(a) the value of d,

(3)

(b) the value of a,

(2)

(c) the total number of houses built in Oldtown over the 40-year period.

(3)
(Total 8 marks)

In the first month after opening, a mobile phone shop sold 280 phones. A model for future trading assumes that sales will increase by x phones per month for the next 35 months, so that $(280 + x)$ phones will be sold in the second month, $(280 + 2x)$ in the third month, and so on.

Using this model with $x = 5$, calculate

(a) (i) the number of phones sold in the 36th month,

(2)

(ii) the total number of phones sold over the 36 months.

(2)

The shop sets a sales target of 17 000 phones to be sold over the 36 months.

Using the same model,

(b) find the least value of x required to achieve this target.

(4)

(Total 8 marks)

An ice cream seller expects that the number of sales will increase by the same amount every week from May onwards. 150 ice creams are sold in Week 1 and 166 ice creams are sold in Week 2. The ice cream seller makes a profit of £1.25 for each ice cream sold.

(a) Find the expected profit in Week 10. [3]

(b) In which week will the total expected profits first exceed £5000? [5]

(c) Give two reasons why this model may not be appropriate. [2]

(a) Ben saves his pocket money as follows.

Each week he puts money into his piggy bank (which pays no interest). In the first week he puts in 10p. In the second week he puts in 12p. In the third week he puts in 14p, and so on.

How much money does Ben have in his piggy bank after 25 weeks? [4]

(b) On January 1st Shirley invests £500 in a savings account that pays compound interest at 3% per annum. She makes no further payments into this account. The interest is added on 31st December each year.

(i) Find the number of years after which her investment will first be worth more than £600. [4]

(ii) State an assumption that you have made in answering part **(b)(i)**. [1]

The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(b) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(d) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)
(Total 10 marks)

A car was purchased for £18 000 on 1st January.

On 1st January each following year, the value of the car is 80% of its value on 1st January in the previous year.

(a) Show that the value of the car exactly 3 years after it was purchased is £9216.

(1)

The value of the car falls below £1000 for the first time n years after it was purchased.

(b) Find the value of n.

(3)

An insurance company has a scheme to cover the maintenance of the car. The cost is £200 for the first year, and for every following year the cost increases by 12% so that for the 3rd year the cost of the scheme is £250.88

(c) Find the cost of the scheme for the 5th year, giving your answer to the nearest penny.

(2)

(d) Find the total cost of the insurance scheme for the first 15 years.

(3)
(Total 9 marks)

The third term of a geometric sequence is 324 and the sixth term is 96

(a) Show that the common ratio of the sequence is $\frac{2}{3}$

(2)

(b) Find the first term of the sequence.

(2)

(c) Find the sum of the first 15 terms of the sequence.

(3)

(d) Find the sum to infinity of the sequence.

(2)
(Total 9 marks)

The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(a) Show that $k^2 - 7k - 60 = 0$.

(4)

(b) Hence show that $k = 12$.

(2)

(c) Find the common ratio of this series.

(2)

(d) Find the sum to infinity of this series.

(2)
(Total 10 marks)

A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(a) the 20th term of the series, to 3 decimal places,

(2)

(b) the sum to infinity of the series.

(2)

Given that the sum to k terms of the series is greater than 24.95,

(c) show that $k > \frac{\log 0.002}{\log 0.8}$,

(4)

The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(a) the common ratio,

(2)

(b) the first term,

(2)

(c) the sum of the first 20 terms, giving your answer to the nearest whole number.

(2)
(Total 6 marks)

A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(a) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(b) Show that $n > \dfrac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(c) find the year in which the profit made will first exceed £200 000,

(2)

(d) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)
(Total 9 marks)

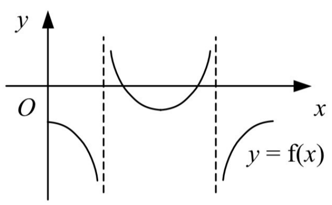
Inverse and reciprocal trigonometric ratios, identities, equations

Sketch each of the following curves for x in the interval $0 \leq x \leq 2\pi$. Show the coordinates of any turning points and the equations of any asymptotes.

a $y = 3 \sec x$ **b** $y = 1 + \operatorname{cosec} x$ **c** $y = \cot 2x$

d $y = \operatorname{cosec}(x - \frac{\pi}{4})$ **e** $y = \sec \frac{1}{3}x$ **f** $y = 3 + 2 \operatorname{cosec} x$

g $y = 1 - \sec 2x$ **h** $y = 2 \cot(x + \frac{\pi}{2})$ **i** $y = 1 + \sec(x - \frac{\pi}{6})$



The diagram shows the curve $y = f(x)$, where

$$f(x) \equiv 2 \cos x - 3 \sec x - 5, \ x \in \mathbb{R}, \ 0 \leq x \leq 2\pi.$$

a Find the coordinates of the point where the curve meets the y-axis.

b Find the coordinates of the points where the curve crosses the x-axis.

$$f(x) \equiv \sin x, \ x \in \mathbb{R}, \ -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}.$$

a State the range of f.

b Define the inverse function $f^{-1}(x)$ and state its domain.

c Sketch on the same diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

$$f(x) \equiv \arccos x - \frac{\pi}{3}, \ x \in \mathbb{R}, -1 \leq x \leq 1.$$

a State the value of $f(-\frac{1}{2})$ in terms of π.

b Solve the equation $f(x) = 0$.

c Define the inverse function $f^{-1}(x)$ and state its domain.

a Solve, in the interval $0 < \theta < 180°$,

i $\operatorname{cosec} \theta = 2 \cot \theta$ **ii** $2 \cot^2 \theta = 7 \operatorname{cosec} \theta - 8$

b Solve, in the interval $0 \leqslant \theta \leqslant 360°$,

i $\sec(2\theta - 15°) = \operatorname{cosec} 135°$ **ii** $\sec^2 \theta + \tan \theta = 3$

c Solve, in the interval $0 \leqslant x \leqslant 2\pi$,

i $\operatorname{cosec}\left(x + \frac{\pi}{15}\right) = -\sqrt{2}$ **ii** $\sec^2 x = \frac{4}{3}$

Prove that:

a $(\tan\theta + \cot\theta)(\sin\theta + \cos\theta) \equiv \sec\theta + \operatorname{cosec}\theta$

b $\dfrac{\operatorname{cosec} x}{\operatorname{cosec} x - \sin x} \equiv \sec^2 x$

c $(1 - \sin x)(1 + \operatorname{cosec} x) \equiv \cos x \cot x$

d $\dfrac{\cot x}{\operatorname{cosec} x - 1} - \dfrac{\cos x}{1 + \sin x} \equiv 2\tan x$

e $\dfrac{1}{\operatorname{cosec}\theta - 1} + \dfrac{1}{\operatorname{cosec}\theta + 1} \equiv 2\sec\theta\tan\theta$

f $\dfrac{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}{1 + \tan^2\theta} \equiv \cos^2\theta$

a Prove that $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} \equiv 2\operatorname{cosec} x$. **(4 marks)**

b Hence solve, in the interval $-2\pi \leqslant x \leqslant 2\pi$, $\dfrac{\sin x}{1 + \cos x} + \dfrac{1 + \cos x}{\sin x} = -\dfrac{4}{\sqrt{3}}$ **(4 marks)**

Prove that $\dfrac{1 + \cos\theta}{1 - \cos\theta} \equiv (\operatorname{cosec}\theta + \cot\theta)^2$ **(4 marks)**

Solve, in the interval $0 \leqslant x \leqslant 2\pi$, the equation $\sec\left(x + \dfrac{\pi}{4}\right) = 2$, giving your answers in terms of π. **(5 marks)**

Solve, in the interval $0 \leqslant x \leqslant 2\pi$, the equation $\sec^2 x - \dfrac{2\sqrt{3}}{3}\tan x - 2 = 0$, giving your answers in terms of π. **(5 marks)**

a Prove that $\sec^4\theta - \tan^4\theta \equiv \sec^2\theta + \tan^2\theta$. **(3 marks)**

b Hence solve, in the interval $-180° \leqslant \theta \leqslant 180°$, $\sec^4\theta = \tan^4\theta + 3\tan\theta$. **(4 marks)**

a Sketch, in the interval $-2\pi \leqslant x \leqslant 2\pi$, the graph of $y = 2 - 3\sec x$. **(3 marks)**

b Hence deduce the range of values of k for which the equation $2 - 3\sec x = k$ has no solutions. **(2 marks)**

Sketch the graphs of:

a $y = \dfrac{\pi}{2} + 2\arcsin x$

b $y = \pi - \arctan x$

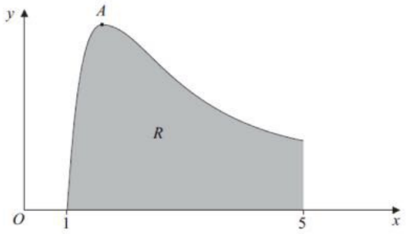
c $y = \arccos(2x + 1)$

d $y = -2\arcsin(-x)$

Area under curves; exponential function

(b) Using integration by parts, find $\int x^{-2} \ln x \, dx$. *(4 marks)*

(c) The sketch shows the graph of $y = x^{-2} \ln x$.

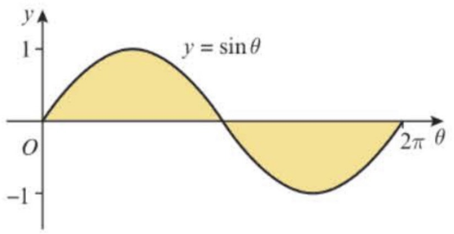


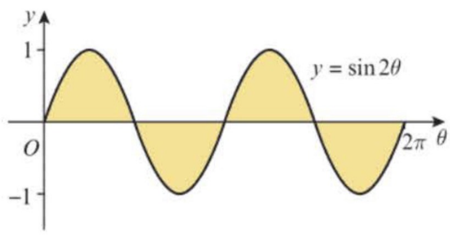
(ii) The region R is bounded by the curve, the x-axis and the line $x = 5$. Using your answer to part (b), show that the area of R is

$$\frac{1}{5}(4 - \ln 5)$$ *(3 marks)*

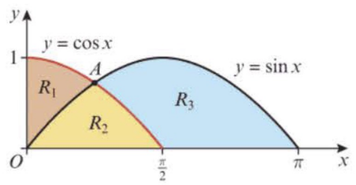
The diagrams show the curves $y = \sin\theta$, $0 \leqslant \theta \leqslant 2\pi$ and $y = \sin 2\theta$, $0 \leqslant \theta \leqslant 2\pi$.

By choosing suitable limits, show that the total shaded area in the first diagram is equal to the total shaded area in the second diagram, and state the exact value of this shaded area.





The diagram shows parts of the graphs of $y = \sin x$ and $y = \cos x$.



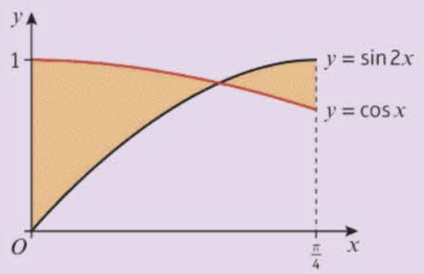
a Find the coordinates of point A.

b Find the areas of:

i R_1 **ii** R_2 **iii** R_3

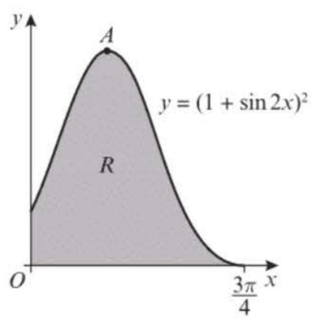
c Show that the ratio of areas $R_1 : R_2$ can be written as $\sqrt{2} : 2$.

The diagram shows the curves $y = \sin 2x$ and $y = \cos x$, $0 \leqslant x \leqslant \frac{\pi}{4}$



Find the exact value of the total shaded area on the diagram.

The diagram shows the graph of $y = (1 + \sin 2x)^2$, $0 \leqslant x \leqslant \frac{3\pi}{4}$

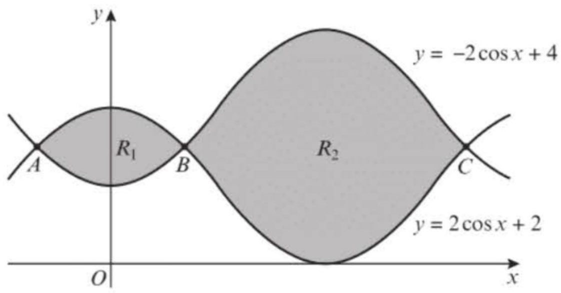


a Show that $(1 + \sin 2x)^2 \equiv \frac{1}{2}(3 + 4\sin 2x - \cos 4x)$. **(4 marks)**

b Hence find the area of the shaded region R. **(4 marks)**

c Find the coordinates of A, the turning point on the graph. **(3 marks)**

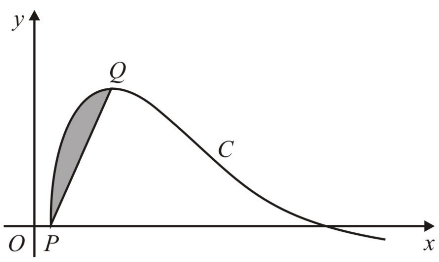
The diagram shows a sketch of part of the curves with equations $y = 2\cos x + 2$ and $y = -2\cos x + 4$.



a Find the coordinates of the points A, B and C. **(2 marks)**

b Find the area of region R_1 in the form $a\sqrt{3} + \frac{b\pi}{c}$, where a, b and c are integers to be found. **(4 marks)**

c Show that the ratio of $R_2 : R_1$ can be expressed as $(3\sqrt{3} + 2\pi) : (3\sqrt{3} - \pi)$. **(5 marks)**



The figure above shows a sketch of part of the curve C with equation

$$y = \sin(\ln x), \qquad x \geq 1.$$

The point Q, on C, is a maximum.

(a) Show that the point $P(1, 0)$ lies on C.

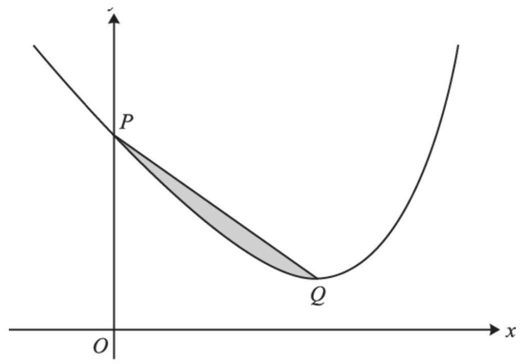
(1)

(b) Find the coordinates of the point Q.

(5)

(c) Find the area of the shaded region between C and the line PQ.

(9)
(Total 15 marks)



The diagram shows the curve

$$y = e^{2x} - 18x + 15.$$

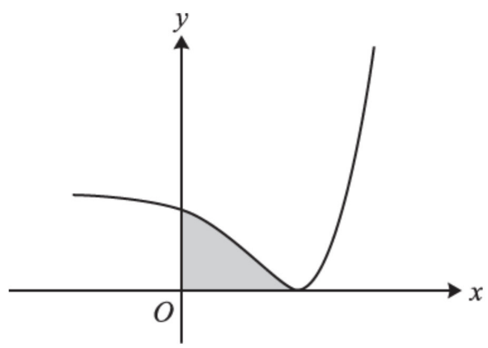
The curve crosses the y-axis at P and the minimum point is Q. The shaded region is bounded by the curve and the line PQ.

i. Show that the x-coordinate of Q is ln3.

[3]

ii. Find the exact area of the shaded region.

[8]



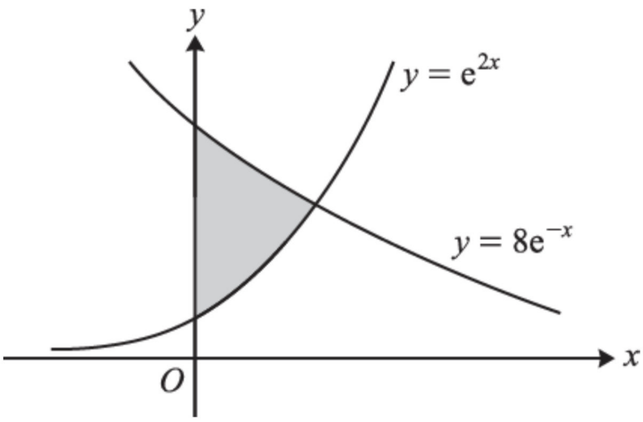
The diagram shows the curve $y = e^{3x} - 6e^{2x} + 32$.

i. Find the exact x-coordinate of the minimum point and verify that the y-coordinate of the minimum point is 0.

[4]

ii. Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes.

[4]



The diagram shows the curves $y = e^{2x}$ and $y = 8e^{-x}$. The shaded region is bounded by the curves and the y-axis. Without using a calculator,

i. solve an appropriate equation to show that the curves intersect at a point for which $x = \ln 2$,

[2]

ii. find the area of the shaded region, giving your answer in simplified form.

[5]

Kinematics graphs

I. The speed of a 100 metre runner in ms^{-1} is measured electronically every 4 seconds.

The measurements are plotted as points on the speed-time graph in Fig. 6. The vertical dotted line is drawn through the runner's finishing time.

Fig. 6 also illustrates Model P in which the points are joined by straight lines.

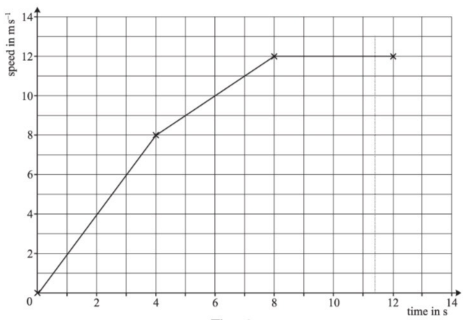


Fig. 6

i. Use Model P to estimate
   A. the distance the runner has gone at the end of 12 seconds,
   B. how long the runner took to complete 100 m.

[6]

A mathematician proposes Model Q in which the runner's speed, $v\,ms^{-1}$ at time t s, is given by

$$v = \frac{5}{2}t - \frac{1}{8}t^2.$$

ii. Verify that Model Q gives the correct speed for $t = 8$.

[1]

iii. Use Model Q to estimate the distance the runner has gone at the end of 12 seconds.

[4]

iv. The runner was timed at 11.35 seconds for the 100 m.

Which model places the runner closer to the finishing line at this time?

[3]

v. Find the greatest acceleration of the runner according to each model.

Fig. 1 shows the velocity-time graph of a cyclist travelling along a straight horizontal road between two sets of traffic lights. The velocity, v, is measured in metres per second and the time, t, in seconds. The distance travelled, s metres, is measured from when $t = 0$.

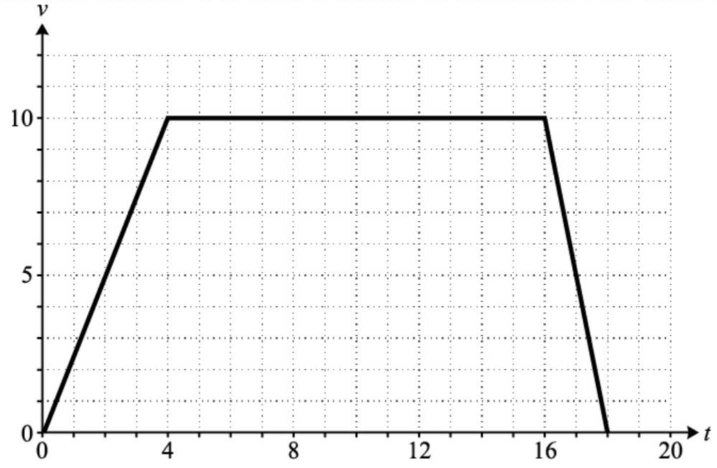


Fig. 1

i. Find the values of s when $t = 4$ and when $t = 18$.

[3]

ii. Sketch the graph of s against t for $0 \leqslant t \leqslant 18$.

A car is usually driven along the whole of a 5 km stretch of road at a constant speed of 25 m s^{-1}. On one occasion, during a period of 50 seconds the speed of the car is as shown by the speed-time graph in Fig. 7; the rest of the 5 km is travelled at 25 m s^{-1}.

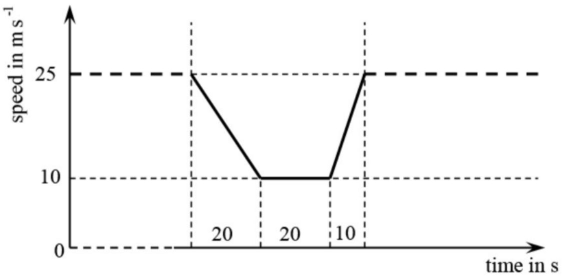


Fig. 7

How much more time than usual did the journey take on this occasion? Show your working clearly. [4]

A bus travelling on a straight road accelerates uniformly from 2.5 m s^{-1} to 7.5 m s^{-1} in 12 s. It then travels at 7.5m s^{-1} for 20 s before slowing uniformly to rest in 8 s.

(a) Sketch a velocity-time graph for the bus. [3]

(b) Calculate the average speed of the bus. [4]

A train is travelling along a straight test track. It starts from rest and reaches its maximum speed after a time of
2 minutes and 21 seconds. During that time it travels 5 km.

Two models, A and B, are considered for its motion.

In Model A, it is assumed that the train has constant acceleration.

(i) Find the acceleration of the train and its maximum speed according to Model A. [5]

In Model B, it is assumed that the acceleration, a m s^{-2} at time t seconds after starting, is given by

$$a = 0.6 - 3 \times 10^{-5} \times t^2.$$

(ii) Show that, according to Model B, the time taken for the train to reach its maximum speed is 2 minutes 21.42 seconds (to the nearest 0.01 s). [2]

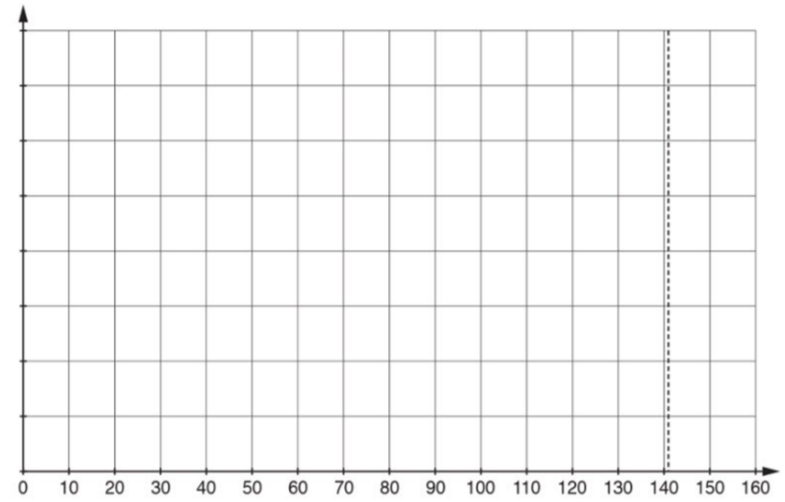
(iii) Find expressions for the speed of the train and the distance that it has travelled at time t, according to Model B. [4]

(iv) Hence show that Model B is consistent with the train travelling a distance of 5 km to attain maximum speed.

Find the maximum speed of the train according to this model. [3]

(v) When the train reaches its maximum speed it continues at that speed.

Draw the speed-time graphs for both models on the grid provided, labelling them A and B. [4]



Rory runs a distance of 45 m in 12.5 s. He starts from rest and accelerates to a speed of 4 m s^{-1}. He runs the remaining distance at 4 m s^{-1}.

Rory proposes a model in which the acceleration is constant until time *T* seconds.

(a) Sketch the velocity-time graph for Rory's run using this model. [2]

(b) Calculate *T*. [2]

(c) Find an expression for Rory's displacement at time *t* s for $0 \leq t \leq T$. [2]

(d) Use this model to find the time taken for Rory to run the first 4 m. [1]

Rory proposes a refined model in which the velocity during the acceleration phase is a quadratic function of *t*. The graph of Rory's quadratic goes through (0, 0) and has its maximum point at (*S*, 4). In this model the acceleration phase lasts until time *S* seconds, after which the velocity is constant.

(e) Sketch a velocity-time graph that represents Rory's run using this refined model. [1]

(f) State with a reason whether *S* is greater than *T* or less than *T*. (You are not required to calculate the value of *S*.) [1]

Kinematics in 2 dimensions using vectors

A particle moves in a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively. At time *t* seconds, its position vector, **r** metres, is given by

$$\mathbf{r} = (2t^3 - t^2 + 6)\mathbf{i} + (8 - 4t^3 + t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the velocity of the particle when $t = \frac{1}{3}$. *(2 marks)*

(ii) State the direction in which the particle is travelling at this time. *(1 mark)*

(c) Find the acceleration of the particle when $t = 4$. *(3 marks)*

(d) The mass of the particle is 6 kg. Find the magnitude of the resultant force on the particle when $t = 4$. *(3 marks)*

Tom is on a fairground ride.

Tom's position vector, **r** metres, at time *t* seconds is given by

$$\mathbf{r} = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (10 - 0.4t)\mathbf{k}$$

The perpendicular unit vectors **i** and **j** are in the horizontal plane and the unit vector **k** is directed vertically upwards.

(a) (i) Find Tom's position vector when $t = 0$. *(1 mark)*

(ii) Find Tom's position vector when $t = 2\pi$. *(1 mark)*

(iii) Write down the first **two** values of *t* for which Tom is directly below his starting point. *(2 marks)*

(b) Find an expression for Tom's velocity at time t. *(3 marks)*

(c) Tom has mass 25 kg.

Show that the resultant force acting on Tom during the motion has constant magnitude. State the magnitude of the resultant force. *(5 marks)*

A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j})\,\mathrm{m\,s^{-1}}$. The velocity of the particle at time t is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

A particle moves in a straight line and at time t it has velocity v, where

$$v = 3t^2 - 2\sin 3t + 6$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. *(2 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

A particle moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively. At time t seconds, the position vector, $\mathbf{r}$ metres, of the particle is given by

$$\mathbf{r} = (t^3 - 3t^2 + 4)\mathbf{i} + (4t + t^2)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) The mass of the particle is 3 kg.

(i) Find an expression for $\mathbf{F}$ at time t. *(3 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 3$. *(2 marks)*

(c) Find the value of t when $\mathbf{F}$ acts due north. *(2 marks)*

A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

(c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

A particle moves on a horizontal plane, in which the unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

At time t seconds, the position vector of the particle is $\mathbf{r}$ metres, where

$$\mathbf{r} = \left(2e^{\frac{1}{2}t} - 8t + 5\right)\mathbf{i} + (t^2 - 6t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the speed of the particle when $t = 3$. *(2 marks)*

(ii) State the direction in which the particle is travelling when $t = 3$. *(1 mark)*

(c) Find the acceleration of the particle when $t = 3$. *(3 marks)*

(d) The mass of the particle is 7 kg.

Find the magnitude of the resultant force on the particle when $t = 3$. *(3 marks)*

A particle moves so that at time t seconds its velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ is given by

$$\mathbf{v} = (4t^3 - 12t + 3)\mathbf{i} + 5\mathbf{j} + 8t\mathbf{k}$$

(a) When $t = 0$, the position vector of the particle is $(-5\mathbf{i} + 6\mathbf{k})$ metres.

Find the position vector of the particle at time t. *(4 marks)*

(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) Find the magnitude of the acceleration of the particle at time t. Do not simplify your answer. *(2 marks)*

(d) Hence find the time at which the magnitude of the acceleration is a minimum. *(2 marks)*

(e) The particle is moving under the action of a single variable force $\mathbf{F}$ newtons. The mass of the particle is 7 kg.

Find the minimum magnitude of $\mathbf{F}$. *(2 marks)*

A particle has mass 200 kg and moves on a smooth horizontal plane. A single horizontal force, $\left(400\cos\left(\frac{\pi}{2}t\right)\mathbf{i} + 600t^2\mathbf{j}\right)$ newtons, acts on the particle at time t seconds.

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) When $t = 4$, the velocity of the particle is $(-3\mathbf{i} + 56\mathbf{j})\,\mathrm{m\,s^{-1}}$.

Find the velocity of the particle at time t. *(5 marks)*

(c) Find t when the particle is moving due west. *(3 marks)*

(d) Find the speed of the particle when it is moving due west. *(2 marks)*

The velocity of a particle at time t seconds is $\mathbf{v}\,\mathrm{m\,s^{-1}}$, where

$$\mathbf{v} = (4 + 3t^2)\mathbf{i} + (12 - 8t)\mathbf{j}$$

(a) When $t = 0$, the particle is at the point with position vector $(5\mathbf{i} - 7\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(4 marks)*

(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) The particle has mass 2 kg.

Find the magnitude of the force acting on the particle when $t = 1$. *(4 marks)*

;

A particle moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons. The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed east and north respectively. At time t seconds, the velocity of the particle, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, is given by

$$\mathbf{v} = 4e^{-2t}\,\mathbf{i} + (6t - 3t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 5 kg.

(i) Find an expression for the force $\mathbf{F}$ acting on the particle at time t. *(2 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 0$. *(2 marks)*

(c) Find the value of t when $\mathbf{F}$ acts due west. *(2 marks)*

(d) When $t = 0$, the particle is at the point with position vector $(6\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

A particle moves in a horizontal plane. The vectors $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors in the horizontal plane. At time t seconds, the velocity of the particle, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, is given by

$$\mathbf{v} = 12\cos\left(\frac{\pi}{3}t\right)\mathbf{i} - 9t^2\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(b) The particle, which has mass 4 kg, moves under the action of a single force, $\mathbf{F}$ newtons.

(i) Find an expression for the force $\mathbf{F}$ in terms of t. *(2 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 3$. *(2 marks)*

(c) When $t = 3$, the particle is at the point with position vector $(4\mathbf{i} - 2\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

A particle, of mass 10 kg, moves on a smooth horizontal plane. At time t seconds, the acceleration of the particle is given by

$$\{(40t + 3t^2)\,\mathbf{i} + 20e^{-4t}\,\mathbf{j}\}\ \mathrm{m\,s^{-2}}$$

where the vectors **i** and **j** are perpendicular unit vectors.

(a) At time $t = 1$, the velocity of the particle is $(6\mathbf{i} - 5e^{-4}\mathbf{j})\ \mathrm{m\,s^{-1}}$.

Find the velocity of the particle at time t. *(5 marks)*

(b) Calculate the initial speed of the particle. *(3 marks)*

The map of a large area of open land is marked in 1 km squares and a point near the middle of the area is defined to be the origin. The vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are in the directions east and north.

At time t hours the position vectors of two hikers, Ashok and Kumar, are given by:

Ashok $\mathbf{r}_A = \begin{pmatrix}-2\\0\end{pmatrix} + \begin{pmatrix}8\\1\end{pmatrix}t,$

Kumar $\mathbf{r}_K = \begin{pmatrix}7t\\10-4t\end{pmatrix}.$

i. Prove that the two hikers meet and give the coordinates of the point where this happens.

[4]

ii. Compare the speeds of the two hikers.

[illegible]

The position vector **r** metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\,\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(a) Find an expression for the velocity of the particle at time t. [2]

(b) Determine whether the particle is ever stationary. [2]

A model boat has velocity v = ((2t – 2)**i** + (2t + 2) **j**) m s $^{-1}$ for $t \geq 0$, where t is the time in seconds.
i is the unit vector east and **j** is the unit vector north.
When $t = 3$, the position vector of the boat is (3**i** + 14**j**) m.

(a) Show that the boat is never instantaneously at rest. [2]

(b) Determine any times at which the boat is moving directly northwards. [2]

(c) Determine any times at which the boat is north-east of the origin. [5]

A toy car moves on a horizontal surface. Its velocity in m s^{-1} is given by

$$\mathbf{v} = 1.5\mathbf{i} + 0.5t\mathbf{j}$$

where **i** and **j** are unit vectors east (x-direction) and north (y-direction) respectively and t is the time in seconds.

Initially the car is at the point 2 m north of the origin.

(a) Calculate the speed of the car after 3 seconds. [2]

(b) Find the position vector of the car after t seconds. [3]

(c) Show that the cartesian equation of the path of the car is $y = \frac{x^2}{9} + 2$. [3]

In this question, $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$ are unit vectors in the x- and y-directions.

A bird is flying in the vertical plane defined by these directions.

The origin is a point on the ground.

The position vector, **r** m, of the bird at time t seconds, where $t \geq 0$, is given by

$$\mathbf{r} = \begin{pmatrix}0\\8\end{pmatrix} + \begin{pmatrix}2\\-4\end{pmatrix}t + \begin{pmatrix}0\\1\end{pmatrix}t^2.$$

(i) Find the velocity of the bird when $t = 2.5$. [3]

(ii) Find the time at which the speed of the bird is 10 m s^{-1}. [3]

(iii) Find the times at which the bird is flying at an angle of 45° to the horizontal. [2]

In this question the unit vectors **i** and **j** are directed east and north respectively.

A model boat sails from a point A with an initial velocity of $-2\mathbf{j}$ m s^{-1}. It accelerates uniformly to a velocity of
$(4\mathbf{i} + 6\mathbf{j})$ms^{-1} in 8 s.

(a) Calculate the acceleration of the boat. [2]

(b) Find the time at which the boat is sailing due east. [2]

4 s after leaving A, the boat is at point B with position vector $(5\mathbf{i} - 2\mathbf{j})$m.

(c) Find the position vector of A. [4]

In this question the positive x and y directions are east and north respectively.
A model boat sails from the origin with initial velocity 3ms^{-1} due west and moves with acceleration

$\begin{pmatrix} -0.1 \\ 0.2 \end{pmatrix}$ ms^{-2} for 25 s.

(a) Show that the velocity of the boat after 25 s is $\begin{pmatrix} -5.5 \\ 5 \end{pmatrix}$ ms^{-1}. [3]

(b) Find the cartesian equation of the path of the boat. [3]

Projectiles

Ali is throwing flat stones onto water, hoping that they will bounce, as illustrated in Fig. 5.

Ali throws one stone from a height of 1.225 m above the water with initial speed 20 ms^{-1} in a horizontal direction. Air resistance should be neglected.

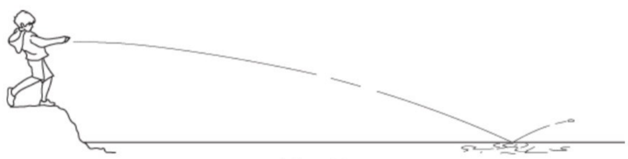


Fig. 5

i. Find the time it takes for the stone to reach the water.

[2]

ii. Find the speed of the stone when it reaches the water and the angle its trajectory makes with the horizontal at this time.

[5]

Arjun is trying to hit a can with a stone. The can is standing on a narrow wall 4 m away from him. The can is
10 cm tall and its base is 1.9 m above the ground, which is level. Arjun throws the stone at the can with a speed of 8 ms^{-1} at an angle of 35° above the horizontal. The point of projection is 1 m above the ground.

Determine whether the stone hits the can. [7]

Fig. 4 illustrates a situation in which a film is being made. A cannon is fired from the top of a vertical cliff towards a ship out at sea. The director wants the cannon ball to fall just short of the ship so that it appears to be a near-miss. There are actors on the ship so it is important that it is not hit by mistake.

The cannon ball is fired from a height 75 m above the sea with an initial velocity of 20ms^{-1} at an angle of 30° above the horizontal. The ship is 90 m from the bottom of the cliff.

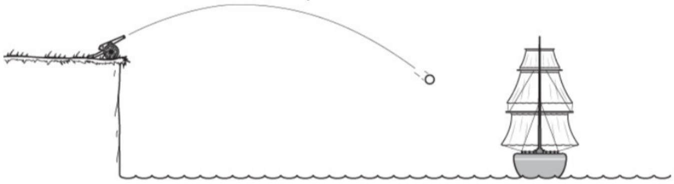


Fig. 4

i. The director calculates where the cannon ball will hit the sea, using the standard projectile model and taking the value of g to be 10ms^{-2}.

Verify that according to this model the cannon ball is in the air for 5 seconds.Show that it hits the water less than 5 m from the ship.

[6]

ii. Without doing any further calculations state, with a brief reason, whether the cannon ball would be predicted to travel further from the cliff if the value of g were taken to be 9.8 ms^{-2}.

In this question, air resistance should be neglected.

Fig. 2 illustrates the flight of a golf ball. The golf ball is initially on the ground, which is horizontal.

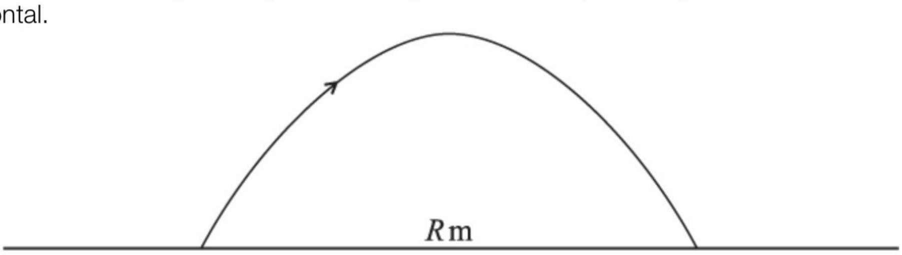


Fig.2

It is hit and given an initial velocity with components of 15 ms^{-1} in the horizontal direction and 20 ms^{-1} in the vertical direction.

i. Find its initial speed.

[1]

ii. Find the ball's flight time and range, R m.

[4]

iii.

A. Show that the range is the same if the components of the initial velocity of the ball are 20 ms^{-1} in the horizontal direction and 15 ms^{-1} in the vertical direction.

[1]

B. State, justifying your answer, whether the range is the same whenever the ball is hit with the same initial speed.

[2]

A golf ball is hit at an angle of 60° to the horizontal from a point, O, on level horizontal ground. Its initial speed is 20 m s^{-1}. The standard projectile model, in which air resistance is neglected, is used to describe the subsequent motion of the golf ball. At time *t* s the horizontal and vertical components of its displacement from O are denoted by *x* m and *y* m.

i. Write down equations for *x* and *y* in terms of *t*.

[2]

ii. Hence show that the equation of the trajectory is

$$y = \sqrt{3}x - 0.049x^2.$$

[2]

iii. Find the range of the golf ball.

[2]

iv. A bird is hovering at position (20,16).

Find whether the golf ball passes above it, passes below it or hits it.

Mr McGregor is a keen vegetable gardener. A pigeon that eats his vegetables is his great enemy.

One day he sees the pigeon sitting on a small branch of a tree. He takes a stone from the ground and throws it. The trajectory of the stone is in a vertical plane that contains the pigeon. The same vertical plane intersects the window of his house. The situation is illustrated in Fig. 5.

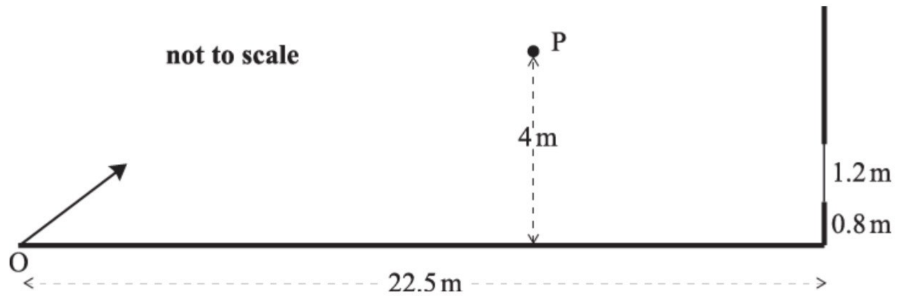


Fig. 5

- The stone is thrown from point O on level ground. Its initial velocity is 15 ms^{-1} in the horizontal direction and 8 ms^{-1} in the vertical direction.
- The pigeon is at point P which is 4 m above the ground.
- The house is 22.5 m from O.
- The bottom of the window is 0.8 m above the ground and the window is 1.2 m high.

Show that the stone does not reach the height of the pigeon.

Determine whether the stone hits the window.

In this question take $g = 10$.

A small stone is projected from a point O with a speed of 26 ms $^{-1}$ at an angle θ above the horizontal. The initial velocity and part of the path of the stone are shown in Fig. 7. You are given that $\sin\theta = \frac{12}{13}$. After t seconds the horizontal displacement of the stone from O is x metres and the vertical displacement is y metres.

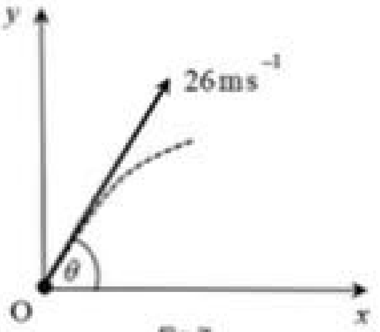


Fig.7

(a) Using the standard model for projectile motion,

- show that $y = 24t - 5t^2$,
- find an expression for x in terms of t. [4]

The stone passes through point A. Point A is 16m above the level of O.

(b) Find the two possible horizontal distances of A from O. [4]

A toy balloon is projected from O with the same initial velocity as the small stone.

(c) Suggest two ways in which the standard model could be adapted. [2]

In this question, **i** is a horizontal unit vector and **j** is a unit vector directed vertically upwards.

A particle is projected from the origin with an initial velocity of $(u_1\mathbf{i} + u_2\mathbf{j})$ms $^{-1}$, and moves freely under gravity. Its position vector **r** m at time t s is given by

$$\mathbf{r} = (u_1\mathbf{i} + u_2\mathbf{j})\ t - 5t^2\mathbf{j}.$$

(a) Write down the value of g used in this model. [1]

(b) Explain what is meant by the statement that g is not a universal constant. [1]

The position vector of the particle when it reaches its maximum height is $(14\mathbf{i} + 20\mathbf{j})$ m.

(c) Determine the initial velocity of the particle, giving your answer as a vector. [7]

(d) The particle hits a building which is 21 m away from the origin in the **i** direction.
Calculate the height above the level of the origin at which the particle hits the building. [3]

In this question you should use the standard projectile model with $g = 9.8$ ms^{-2}.

Fig. 7 illustrates the trajectory of a tennis ball which has been served by a player. It is not drawn to scale.

- The ball must pass over the net and land in the service court.
- The player hits the ball at an angle of α above the horizontal.

Three junior members of a tennis club take turns to serve a tennis ball. They are Hamish (a beginner), Oscar (of medium standard) and Tara (a good player). They each stand at the same point and hit the ball in the same vertical plane at the same point P. The following figures apply to their serves.

- The player hits the ball from a height of 2.22 m.
- The height of the net is 0.995 m.
- The player is 12.5 m from the net.
- The ball must bounce within 6.5 m of the net.

Fig. 7

Hamish serves the ball with components of velocity 10 m s^{-1} horizontally and 5.5 m s^{-1} vertically upwards.

(i) Find the speed of Hamish's serve and the value of α. [2]

(ii) Show that Hamish's serve passes over the net. [3]

(iii) Find the time at which Hamish's serve hits the ground.

Does it land in the service court? [4]

Oscar hits the ball horizontally, so $\alpha = 0$. The initial speed of the ball is u m s^{-1}.

(iv) Find the range of possible values of u for which the ball lands in the service court. [6]

Tara serves the ball at an angle of 2° below the horizontal. The ball clears the net and bounces after 0.57 seconds.

(v) Find the initial speed of Tara's serve. [3]

A goalkeeper kicks a football from ground level on a level playing field. The ball is in the air for 3.5 s.

(a) State a modelling assumption in the standard projectile model. [1]

(b) Calculate the vertical component of the initial velocity of the ball. [2]

(c) Calculate the maximum height of the ball. [2]

(d) The ball lands 42 m from its original position. Calculate [3]

(i) the initial speed of the ball,

(ii) the angle that the initial velocity makes with the ground. [2]

A pebble is thrown horizontally at 14 m s^{-1} from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

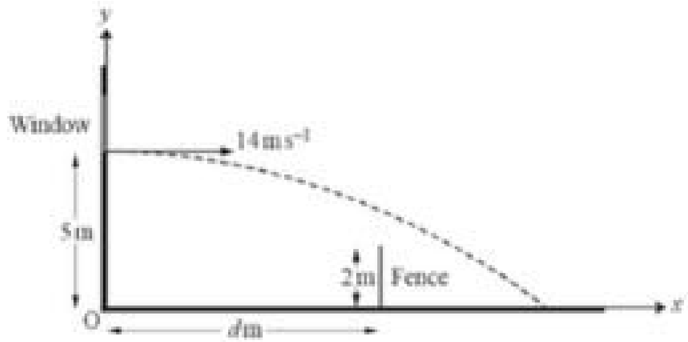


Fig. 9

(a) Find the time the pebble takes to reach the ground. [3]

(b) Find the cartesian equation of the trajectory of the pebble. [4]

(c) Find the range of possible values for d. [3]

A pebble is thrown horizontally at 21 m s^{-1} from a point 1.6 m above level ground. Calculate the horizontal distance travelled by the pebble before it hits the ground.

[4]

Forces, friction

A trolley, of mass 100 kg, rolls at a constant speed along a straight line down a slope inclined at an angle of 4° to the horizontal.

Assume that a constant resistance force, of magnitude P newtons, acts on the trolley as it moves. Model the trolley as a particle.

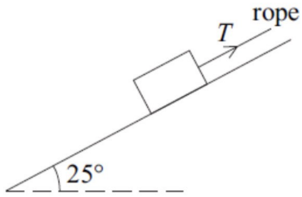
(a) Draw a diagram to show the forces acting on the trolley. *(1 mark)*

(b) Show that $P = 68.4$ N, correct to three significant figures. *(3 marks)*

(c) (i) Find the acceleration of the trolley if it rolls down a slope inclined at 5° to the horizontal and experiences the same constant force of magnitude P that you found in part (b). *(4 marks)*

(ii) Make one criticism of the assumption that the resistance force on the trolley is constant. *(1 mark)*

A rough slope is inclined at an angle of 25° to the horizontal. A box of weight 80 newtons is on the slope. A rope is attached to the box and is parallel to the slope. The tension in the rope is of magnitude T newtons. The diagram shows the slope, the box and the rope.



(a) The box is held in equilibrium by the rope.

(i) Show that the normal reaction force between the box and the slope is 72.5 newtons, correct to three significant figures. *(3 marks)*

(ii) The coefficient of friction between the box and the slope is 0.32. Find the magnitude of the maximum value of the frictional force which can act on the box. *(2 marks)*

(iii) Find the least possible tension in the rope to prevent the box from moving down the slope. *(4 marks)*

(iv) Find the greatest possible tension in the rope. *(3 marks)*

(v) Show that the mass of the box is approximately 8.16 kg. *(1 mark)*

(b) The rope is now released and the box slides down the slope. Find the acceleration of the box. *(3 marks)*

A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

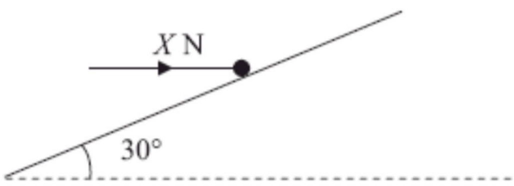
(a) the acceleration of the particle,

(3)

(b) the coefficient of friction between the particle and the plane.

(5)

The particle is now held on the same rough plane by a horizontal force of magnitude x newtons, acting in a plane containing a line of greatest slope of the plane, as shown in the diagram above. The particle is in equilibrium and on the point of moving up the plane.

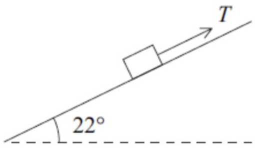


(c) Find the value of X.

(7)
(Total 15 marks)

A block is being pulled up a rough plane inclined at an angle of 22° to the horizontal by a rope parallel to the plane, as shown in the diagram.

The mass of the block is 0.7 kg, and the tension in the rope is T newtons.



(a) Draw a diagram to show the forces acting on the block. *(1 mark)*

(b) Show that the normal reaction force between the block and the plane has magnitude 6.36 newtons, correct to three significant figures. *(3 marks)*

(c) The coefficient of friction between the block and the plane is 0.25. Find the magnitude of the frictional force acting on the block during its motion. *(2 marks)*

(d) The tension in the rope is 5.6 newtons. Find the acceleration of the block. *(4 marks)*

A box, of mass 3 kg, is placed on a slope inclined at an angle of 30° to the horizontal. The box slides down the slope. Assume that air resistance can be ignored.

(a) A simple model assumes that the slope is smooth.

(i) Draw a diagram to show the forces acting on the box. *(1 mark)*

(ii) Show that the acceleration of the box is $4.9\ \text{m s}^{-2}$. *(2 marks)*

(b) A revised model assumes that the slope is rough. The box slides down the slope from rest, travelling 5 metres in 2 seconds.

(i) Show that the acceleration of the box is $2.5\ \text{m s}^{-2}$. *(2 marks)*

(ii) Find the magnitude of the friction force acting on the box. *(3 marks)*

(iii) Find the coefficient of friction between the box and the slope. *(5 marks)*

(iv) In reality, air resistance affects the motion of the box. Explain how its acceleration would change if you took this into account. *(2 marks)*

Pulley, rough surface, Newton's Laws

Fig. 3.1 shows a block of mass 8 kg on a smooth horizontal table.

This block is connected by a light string passing over a smooth pulley to a block of mass 4 kg which hangs freely. The part of the string between the 8 kg block and the pulley is parallel to the table.

The system has acceleration $a\ \text{ms}^{-2}$.

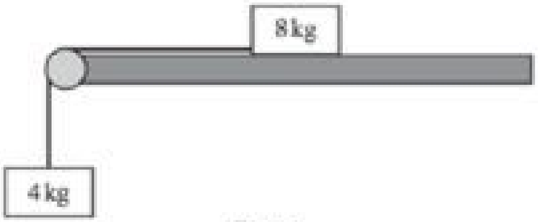


Fig. 3.1

i. Write down two equations of motion, one for each block.

ii. Find the value of a.

The table is now tilted at an angle of θ to the horizontal as shown in Fig. 3.2. The system is set up as before: the 4 kg block still hangs freely.

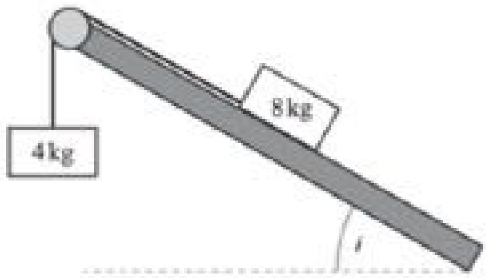
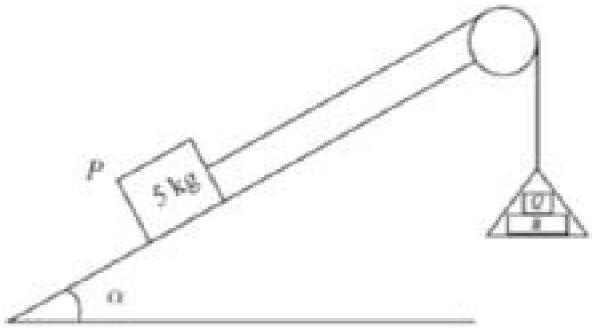


Fig. 3.2

iii. The system is now in equilibrium. Find the value of θ.



One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R, as shown in Figure 3. The mass of block Q is 5 kg and the mass of block R is 10 kg. The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find

(a) (i) the acceleration of the scale pan,

(ii) the tension in the string,

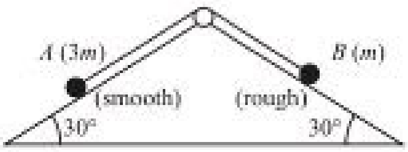
(8)

(b) the magnitude of the force exerted on block Q by block R,

(3)

(c) the magnitude of the force exerted on the pulley by the string.

(5)
(Total 16 marks)



A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles A and B, of mass $3m$ and m respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which A moves is smooth. The face on which B moves is rough. The coefficient of friction between B and this face is μ. Particle A is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in the figure above.

The particles are released from rest and start to move. Particle A moves downwards and B moves upwards. The accelerations of A and B each have magnitude $\frac{1}{10}g$.

(a) By considering the motion of A, find, in terms of m and g, the tension in the string.

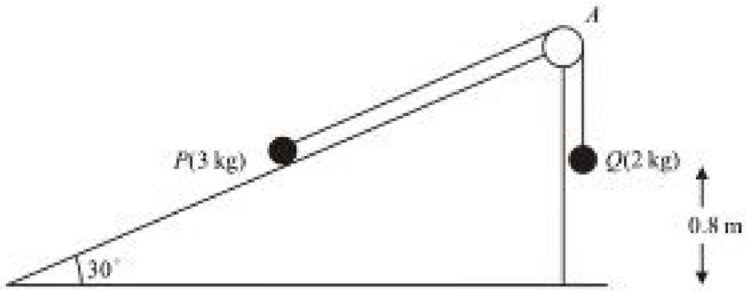
(3)

(b) By considering the motion of B, find the value of μ.

(8)

(c) Find the resultant force exerted by the string on the pulley, giving its magnitude and direction.

(3)
(Total 14 marks)



The diagram above shows two particles P and Q, of mass 3 kg and 2 kg respectively, connected by a light inextensible string. Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth light pulley A fixed at the top of the plane. The part of the string from P to A is parallel to a line of greatest slope of the plane. The particle Q hangs freely below A. The system is released from rest with the string taut.

(a) Write down an equation of motion for P and an equation of motion for Q.

(4)

(b) Hence show that the acceleration of Q is 0.98 m s^{-2}.

(2)

(c) Find the tension in the string.

(2)

(d) State where in your calculations you have used the information that the string is inextensible.

(1)

On release, Q is at a height of 0.8 m above the ground. When Q reaches the ground, it is brought to rest immediately by the impact with the ground and does not rebound. The initial distance of P from A is such that in the subsequent motion P does not reach A. Find

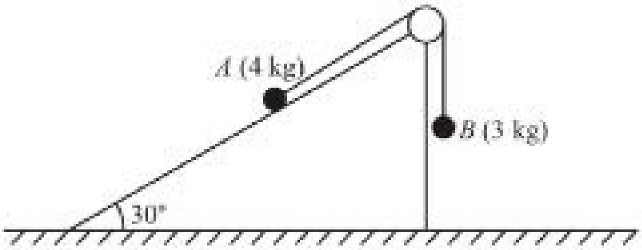
(e) the speed of Q as it reaches the ground,

(2)

(f) the time between the instant when Q reaches the ground and the instant when the string becomes taut again.

(5)

(Total 16 marks)



A particle A of mass 4 kg moves on the inclined face of a smooth wedge. This face is inclined at 30° to the horizontal. The wedge is fixed on horizontal ground. Particle A is connected to a particle B, of mass 3 kg, by a light inextensible string. The string passes over a small light smooth pulley which is fixed at the top of the plane. The section of the string from A to the pulley lies in a line of greatest slope of the wedge. The particle B hangs freely below the pulley, as shown in the diagram above. The system is released from rest with the string taut. For the motion before A reaches the pulley and before B hits the ground, find

(a) the tension in the string,

(6)

(b) the magnitude of the resultant force exerted by the string on the pulley.

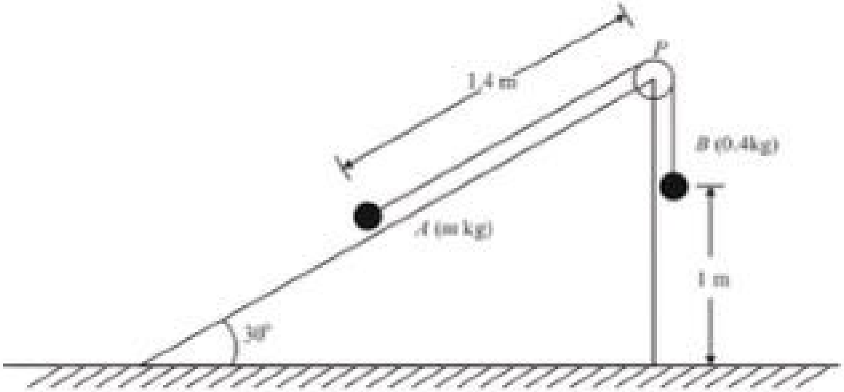
(3)

(c) The string in this question is described as being 'light'.

(i) Write down what you understand by this description.

(ii) State how you have used the fact that the string is light in your answer to part (a).

(2)
(Total 11 marks)



The diagram above shows two particles A and B, of mass m kg and 0.4 kg respectively, connected by a light inextensible string. Initially A is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small light smooth pulley P fixed at the top of the plane. The section of the string from A to P is parallel to a line of greatest slope of the plane. The particle B hangs freely below P. The system is released from rest with the string taut and B descends with acceleration $\frac{1}{5}g$.

(a) Write down an equation of motion for B.

(2)

(b) Find the tension in the string.

(2)

(c) Prove that $m = \frac{16}{35}$.

(4)

(d) State where in the calculations you have used the information that P is a light smooth pulley.

(1)

On release, B is at a height of one metre above the ground and AP = 1.4 m. The particle B strikes the ground and does not rebound.

(e) Calculate the speed of B as it reaches the ground.

(2)

(f) Show that A comes to rest as it reaches P.

(5)

(Total 16 marks)

Statics; forces, friction, equilibrium

Two forces, $(4\mathbf{i} - 5\mathbf{j})$ N and $(p\mathbf{i} + q\mathbf{j})$ N, act on a particle P of mass m kg. The resultant of the two forces is $\mathbf{R}$. Given that $\mathbf{R}$ acts in a direction which is parallel to the vector $(\mathbf{i} - 2\mathbf{j})$,

(a) find the angle between $\mathbf{R}$ and the vector $\mathbf{j}$.

(3)

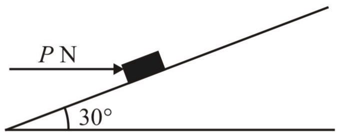
(b) show that $2p + q + 3 = 0$.

(4)

Given also that $q = 1$ and that P moves with an acceleration of magnitude $8\sqrt{5}$ m s^{-2},

(c) find the value of m.

(7)
(Total 14 marks)



A parcel of weight 10N lies on a rough plane inclined at an angle of 30° to the horizontal. A horizontal force of magnitude P newtons acts on the parcel, as shown in the figure above. The parcel is in equilibrium and on the point of slipping up the plane. The normal reaction of the plane on the parcel is 18N. The coefficient of friction between the parcel and the plane is μ. Find

(a) the value of P,

(4)

(b) the value of μ.

(5)

The horizontal force is removed.

(c) Determine whether or not the parcel moves.

(5)
(Total 14 marks)

Fig. 15 shows a uniform shelf AB of weight W N.
The shelf is 180 cm long and rests on supports at points C and D. Point C is 30 cm from A and point D is 60 cm from B.

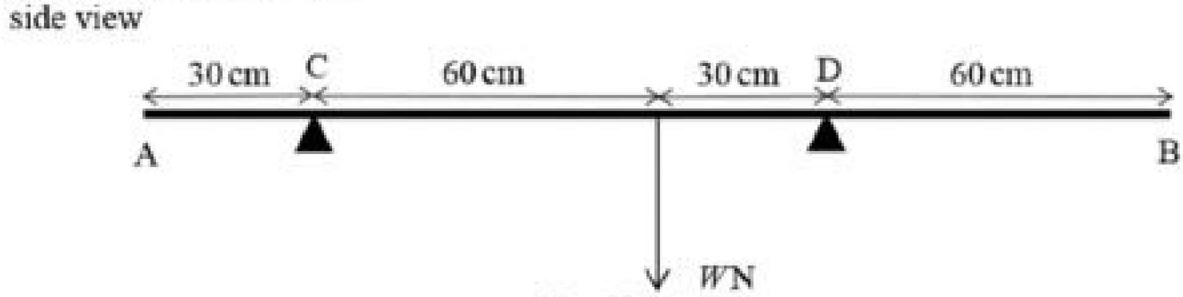


Fig. 15

Determine the range of positions a point load of 3 W could be placed on the shelf without the shelf tipping. [6]

A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

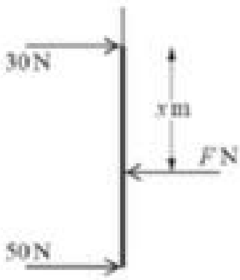


Fig. 7

(a) Find the value of F. [1]

(b) Find the value of x. [2]

Fig. 7 shows a rectangular lamina ABCD with sides AB of length 60 cm and AD of length 50 cm. The lamina is lying flat on a smooth horizontal surface, and is acted on by the following five horizontal forces.

- 45 N at A in the direction BA
- 40 N at D in the direction AD
- 27 N at C in the direction BC
- *Y* N at A in the direction DA
- *X* N at E in a direction perpendicular to the edge BC

The lamina is in equilibrium. The distance BE is *d* cm.

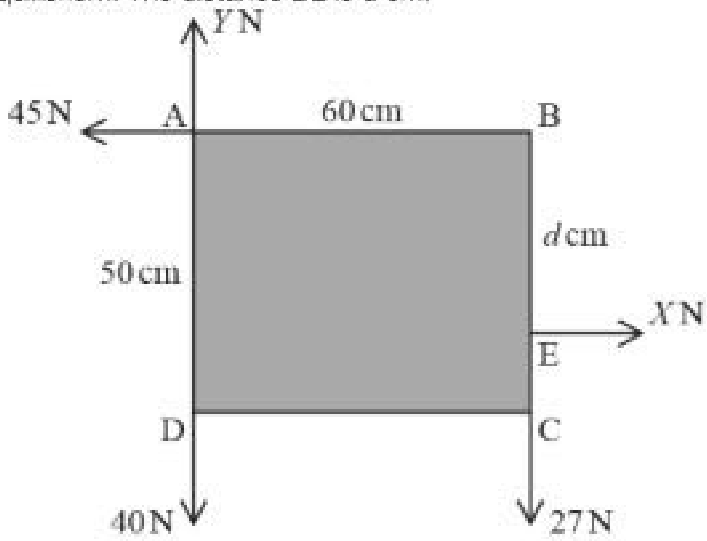


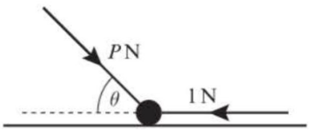
Fig. 7

Calculate the values of each of the following.

- *X*
- *Y*
- *d*

[4]

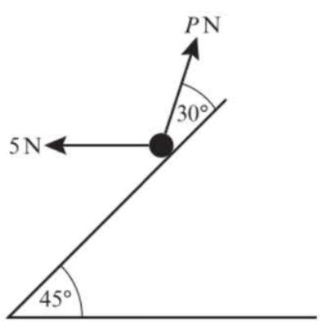
A particle of weight 2 N rests on a smooth horizontal surface and remains in equilibrium under the action of the two external forces shown in the diagram. One is a horizontal force of magnitude 1 N and the other is a force of magnitude *P* N which acts at an angle θ to the horizontal, where $\tan\theta = \frac{12}{5}$. Find:



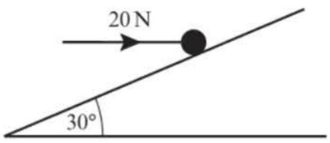
a the value of *P* **(3 marks)**

b the normal reaction between the particle and the surface. **(2 marks)**

A particle of weight 20 N rests in equilibrium on a smooth inclined plane. It is maintained in equilibrium by the application of two external forces as shown in the diagram. One of the forces is a horizontal force of 5 N, the other is a force *P* N acting at an angle of 30° to the plane, as shown in the diagram. Find the magnitude of the normal reaction between the particle and the plane. **(8 marks)**



A horizontal force of magnitude 20 N acts on a block of mass 1.5 kg, which is in equilibrium resting on a rough plane inclined at 30° to the horizontal. The line of action of the force is in the same vertical plane as the line of greatest slope of the inclined plane.



a Find the normal reaction between the block and the plane. **(4 marks)**

b Find the magnitude and direction of the frictional force acting on the block. **(3 marks)**

c Hence find the minimum value of the coefficient of friction between the block and the plane. **(2 marks)**