# Question Examples from Advance Information for Paper 2 (OCR MEI)

## Proof

Prove, by counter-example, that each of the following statements is false.

**a** For all positive real values of $x$, $\sqrt[3]{x} \leq x$. **(2)**

**b** For all positive integer values of $n$, $(n^3 - n + 7)$ is prime. **(2)**

**a** Given that $n = 2m + 1$, find and simplify an expression in terms of $m$ for $n^2 + 2n$. **(1)**

**b** Hence, use proof by contradiction to prove that if $(n^2 + 2n)$ is even, where $n$ is an integer, then $n$ is even. **(5)**

Use proof by contradiction to prove that there are no positive integers, $x$ and $y$, such that

$$x^2 - y^2 = 1.$$ **(6)**

For each statement, either prove that it is true or find a counter-example to prove that it is false.

**a** If $a$ and $b$ are irrational and $a \neq b$, then $(a + b)$ is irrational. **(2)**

**b** If $m$ and $n$ are consecutive odd integers, then $(m + n)$ is divisible by 4. **(3)**

**c** For all real values of $x$, $\cos x \leq 1 + \sin x$. **(2)**

**a** Show that if $\log_2 3 = \frac{p}{q}$, then

$$2^p = 3^q.$$ **(2)**

**b** Use proof by contradiction to prove that $\log_2 3$ is irrational. **(4)**

**c** Prove, by counter-example, that the statement

"if $a$ is rational and $b$ is irrational then $\log_a b$ is irrational"

is false. **(2)**

Use proof by contradiction to prove each of the following statements.

**a** If $n^3$ is odd, where $n$ is a positive integer, then $n$ is odd.

**b** If $x$ is irrational, then $\sqrt{x}$ is irrational.

**c** If $a$, $b$ and $c$ are integers and $bc$ is not divisible by $a$, then $b$ is not divisible by $a$.

**d** If $(n^2 - 4n)$ is odd, where $n$ is a positive integer, then $n$ is odd.

**e** There are no positive integers, $m$ and $n$, such that $m^2 - n^2 = 6$.

Given that $x$ and $y$ are integers and that $(x^2 + y^2)$ is divisible by 4, use proof by contradiction to prove that

**a** $x$ and $y$ are not both odd,

**b** $x$ and $y$ are both even.

**a** Prove that if

$$\sqrt{2} = \frac{p}{q},$$

where $p$ and $q$ are integers, then $p$ must be even.

**b** Use proof by contradiction to prove that $\sqrt{2}$ is irrational.

Prove by contradiction that there is no greatest even positive integer. [3]

Prove algebraically that $n^3 + 3n - 1$ is odd for all positive integers $n$. [4]

Prove that the sum of the squares of any two consecutive integers is of the form $4k + 1$, where $k$ is an integer.

[4]

**a** Prove that the difference of the squares of two consecutive even numbers is always divisible by 4.

**b** Is this statement true for odd numbers? Give a reason for your answer.

Prove by contradiction that $\sqrt{2}$ is an irrational number.

Prove by contradiction that there are infinitely many prime numbers.

Parametric equations